Differential Equations 2

Q1.

The variables x and y are related by the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2x + 1.$$

(a) Find the general solution for y in terms of x. [6]

(b) State an approximate solution for large positive values of x. [1]

Q2.

Find the solution of the differential equation

$$\sin\theta \frac{dy}{d\theta} + y = \tan\tfrac{1}{2}\theta,$$

where $0 < \theta < \pi$, given that $y = 1$ when $\theta = \frac{1}{2}\pi$. Give your answer in the form $y = f(\theta)$. [9]

[You may use without proof the result that $\int \operatorname{cosec}\theta \, d\theta = \ln\tan\tfrac{1}{2}\theta$.]

Q3.

The variables x and y are related by the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 4e^{-x}.$$

(a) Find the value of the constant k such that $y = kxe^{-x}$ is a particular integral of the differential equation. [4]

(b) Find the solution of the differential equation for which $y = \frac{dy}{dx} = \frac{1}{2}$ when $x = 0$. [6]

Q4.

(a) Starting from the definitions of sinh and cosh in terms of exponentials, prove that

$$2\sinh^2 x = \cosh 2x - 1.$$ [3]

(b) Find the solution to the differential equation

$$\frac{dy}{dx} + y\coth x = 4\sinh x$$

for which $y = 1$ when $x = \ln 3$. [7]

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Q5.

Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\cos x,$$

given that, when $x = 0$, $y = -4$ and $\frac{dy}{dx} = 3$. [11]

Q6.

(a) Show that an appropriate integrating factor for

$$\sqrt{x^2-1}\frac{dy}{dx} + y = x^2 - x\sqrt{x^2-1}$$

is $x + \sqrt{x^2-1}$. [4]

(b) Hence find the solution of the differential equation

$$\sqrt{x^2-1}\frac{dy}{dx} + y = x^2 - x\sqrt{x^2-1}$$

for which $y = 1$ when $x = \frac{5}{4}$. Give your answer in the form $y = f(x)$. [7]

Q7.

Find the solution of the differential equation

$$\frac{dy}{dx} + \frac{4x^3y}{x^4+5} = 6x$$

for which $y = 1$ when $x = 1$. Give your answer in the form $y = f(x)$. [7]

Q8.

The variables t and x are related by the differential equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = t^2 + 1.$$

(a) Find the general solution for x in terms of t. [6]

(b) Deduce an approximate value of $\frac{d^2x}{dt^2}$ for large positive values of t. [2]

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Q9.

Use the substitution $y = vx$ to find the solution of the differential equation

$$x\frac{dy}{dx} = y + \sqrt{9x^2 + y^2}$$

for which $y = 0$ when $x = 1$. Give your answer in the form $y = f(x)$, where $f(x)$ is a polynomial in x. [10]

Q10.

Find the solution of the differential equation

$$x(x+7)\frac{dy}{dx} + 7y = x$$

for which $y = 7$ when $x = 1$. Give your answer in the form $y = f(x)$. [9]

Q11.

The variables x and y are related by the differential equation

$$4\frac{d^2y}{dx^2} - y = 3.$$

It is given that, when $x = 0$, $y = -3$ and $\frac{dy}{dx} = 2$.

(a) Find y in terms of x. [8]

(b) Deduce the exact value of x for which $y = 0$. Give your answer in logarithmic form. [3]
