Differentiation 1 MS

Q1.

4	Differentiates with respect to x. Substitutes (–1, 1)	$2y^2 + 4xyy' + 6xy + 3x^2y' = 0$ $2 - 4y' - 6 + 3y' = 0 \Rightarrow y' = -4$ (AG)	B1B1 B1	3	
	Differentiates again.	$4yy' + (4y + 4xy')y' + 4xyy'' + 6y + 6xy'$ $+ 6xy' + 3x^2y'' = 0$	B1B1 B1		
	Substitutes (–1, 1) and $y' = -4$	$-16 - 80 - 4y'' + 6 + 24 + 24 + 3y'' = 0$ $\Rightarrow y'' = -42$	M1 A1	5	**[8]**

Q2.

4	Finds first derivative.	$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{-6\sin 2t}{4\cos 2t} = -\frac{3}{2}\tan 2t$	M1A1		
	Evaluates.	When $t = \frac{\pi}{3}$, $\frac{dy}{dx} = \frac{3\sqrt{3}}{2}$	A1	3	
(ii)	Finds second derivative.	$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx} = -3\sec^2 2t \times \frac{1}{4}\sec 2t$	M1A1		
		$= -\frac{3}{4}\sec^3 2t$	A1		
	Evaluates.	When $t = \frac{\pi}{3}$, $\frac{d^2y}{dx^2} = \frac{3}{4} \times 8 = 6$	A1	4	**[7]**
	Alternatively				
(i)	Finds cartesian equation and differentiates implicitly.	$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1 \Rightarrow y' = -\frac{9x}{4y} = -\frac{9}{4} \times \frac{-2\sqrt{3}}{3} = \frac{3\sqrt{3}}{2}$	M1A1 A1	3	
(ii)	Differentiates again.	$\frac{1}{2} + \frac{2}{9}\left[(y')^2 + yy''\right] = 0 \Rightarrow \frac{1}{2} + \frac{3}{2} = \frac{1}{3}y'' \Rightarrow y'' = 6$	M1A2 A1	4	**[7]**

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Q3.

3		$y + x\frac{dy}{dx} + 3(x+y)^2\left(1+\frac{dy}{dx}\right) = 0$	B1B1		
		$0 + y' + 3 + 3y' = 0$			
		$\Rightarrow y' = -\frac{3}{4}$ (AG)	B1	3	
		$\frac{dy}{dx} + \frac{dy}{dx} + x\frac{d^2y}{dx^2} + 6(x+y)\left(1+\frac{dy}{dx}\right)^2 + 3(x+y)^2\frac{d^2y}{dx^2} = 0$	B1 B1B1		
		$-\frac{3}{4} - \frac{3}{4} + y'' + 6\times\frac{1}{16} + 3y'' = 0$	M1		
		$\Rightarrow y'' = \frac{9}{32}$	A1	5	
		N.B. Mark similarly if expression expanded before differentiating.			**[8]**

Q4.

4	Differentiates each equation wrt θ.	$\dot{x} = 2 - 2\cos 2\theta \quad \dot{y} = 2\sin 2\theta$	B1		
	Forms quotient for $\frac{dy}{dx}$	$\frac{dy}{dx} = \frac{4\sin\theta\cos\theta}{4\sin^2\theta} = \cot\theta$ (AG)	M1A1		
	States exceptions.	Except for $\theta = k\pi$ (k an integer).	A1	**(4)**	
	Differentiates wrt x	$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\cot\theta \times \frac{1}{x} = -\text{cosec}^2\theta \times \frac{1}{4\sin^2\theta}$	M1A1		
	Obtains result	$= -1$ when $\theta = \frac{1}{4}\pi$.	A1	**(3)**	
					[7]

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Q5.

12E (i)	$\dot{x}=2t$, $\dot{y}=-\frac{1}{2}(2-t)^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx}=\frac{-1}{4t(2-t)^{\frac{1}{2}}}$.	M1A1
	$\frac{d^2y}{dx^2}=\left\{4t(2-t)^{\frac{1}{2}}\right\}^{-2}\left\{4(2-t)^{\frac{1}{2}}-\frac{4t}{2(2-t)^{\frac{1}{2}}}\right\}\times\frac{1}{2t}$ (OE)	M1A1
	$=\frac{1}{16t^3(2-t)^{\frac{3}{2}}}(4-3t)$ (Any correct simplified form.)	A1 [5]
(ii)	$\int_0^4 y\,dx=\int_0^2 2t(2-t)^{\frac{1}{2}}\,dt$	
	$=2\left\{\left[-\frac{2}{3}t(2-t)^{\frac{3}{2}}\right]_0^2+\int_0^2\frac{2}{3}(2-t)^{\frac{3}{2}}\,dt\right\}$ (LNR)	M1A1
	$=2\left\{0+\left[-\frac{4}{15}(2-t)^{\frac{5}{2}}\right]_0^2\right\}=\frac{32}{15}\sqrt{2}$ (LR)	M1A1
	$\text{MV}=\frac{\int_0^4 y\,dx}{4-0}=\frac{1}{4}\times\frac{32}{15}\sqrt{2}=\frac{8}{15}\sqrt{2}$ or 0.754.	M1A1 [6]
(iii)	$\frac{1}{2}\int_0^4 y^2\,dx=\frac{1}{2}\int_0^2 2t(2-t)\,dt=\left[t^2-\frac{1}{3}t^3\right]_0^2=\frac{4}{3}$ or 1.33.	M1A1
	$\bar{y}=\frac{\frac{1}{2}\int_0^4 y^2\,dx}{\int_0^4 y\,dx}=\frac{4}{3}\times\frac{15}{32\sqrt{2}}=\frac{5}{8\sqrt{2}}=\frac{5}{16}\sqrt{2}$ or 0.442.	A1 [3]

Q6.

1(i)	$-\sin y\frac{dy}{dx}=1\Rightarrow\frac{dy}{dx}=-(\sin y)^{-1}$	**M1 A1**	Differentiates implicitly once.
	$\frac{d^2y}{d^2x}=(\sin y)^{-2}\cos y\left(\frac{dy}{dx}\right)=-\cot y\left(\frac{dy}{dx}\right)^2$	**M1 A1**	Differentiates again, AG.
		4	
1(ii)	$\frac{dy}{dx}=-\left(\sin\frac{\pi}{3}\right)^{-1}=-\frac{2}{\sqrt{3}}$	**B1**	
	$\frac{d^2y}{d^2x}=-\frac{1}{\sqrt{3}}\left(-\frac{2}{\sqrt{3}}\right)^2=-\frac{4}{3\sqrt{3}}=-\frac{4}{9}\sqrt{3}$	**B1**	AEF, must be exact.
		2	

Differentiation 1 MS

Q7.

2(a)	$\ln y = x\ln 2 \Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln 2$	**M1 A1**
	$\frac{dy}{dx} = y\ln 2 = 2^x \ln 2$ **AG**	**A1**
		3
2(b)	$\frac{d^2y}{dx^2} = 2^x(\ln 2)^2$	**B1**
		1

2(c)	$y(0) = 1, \quad y'(0) = \ln 2, \quad y''(0) = (\ln 2)^2$	**B1**
	$2^x = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 + \ldots = 1 + (\ln 2)x + \frac{(\ln 2)^2}{2}x^2 + \ldots$	**M1 A1**
		3

Q8.

6(a)	$\tanh\theta = \frac{e^{\theta} - e^{-\theta}}{e^{\theta} + e^{-\theta}} \quad \operatorname{sech}\theta = \frac{2}{e^{\theta} + e^{-\theta}}$	**B1**
	$1 - \left(\frac{e^{\theta} - e^{-\theta}}{e^{\theta} + e^{-\theta}}\right)^2 = \frac{(e^{\theta} + e^{-\theta})^2 - (e^{\theta} - e^{-\theta})^2}{(e^{\theta} + e^{-\theta})^2} = \frac{4}{(e^{\theta} + e^{-\theta})^2}$	**M1**
	$= \operatorname{sech}^2\theta$	**A1**
		3

6(b)	$\tanh y = \cos(x + \frac{1}{4}\pi) \quad \Rightarrow \quad \operatorname{sech}^2 y\frac{dy}{dx} = -\sin(x + \frac{1}{4}\pi)$	**M1 A1**
	$\left(1 - \cos^2(x + \frac{1}{4}\pi)\right)\frac{dy}{dx} = -\sin(x + \frac{1}{4}\pi)$	**M1**
	$\frac{dy}{dx} = -\frac{\sin(x + \frac{1}{4}\pi)}{\sin^2(x + \frac{1}{4}\pi)} = -\operatorname{cosec}(x + \frac{1}{4}\pi)$	**A1**
		4
6(c)	$\frac{d^2y}{dx^2} = \cot(x + \frac{1}{4}\pi)\operatorname{cosec}(x + \frac{1}{4}\pi)$	**B1**
	$y'(0) = -\operatorname{cosec}(\frac{1}{4}\pi) \quad y''(0) = \cot(\frac{1}{4}\pi)\operatorname{cosec}(\frac{1}{4}\pi)$	**M1**
	$y(0) = \tanh^{-1}\left(\frac{1}{2}\sqrt{2}\right) = \frac{1}{2}\ln\left(\frac{2+\sqrt{2}}{2-\sqrt{2}}\right)$	**M1**
	$y = \frac{1}{2}\ln(3 + 2\sqrt{2}) - x\sqrt{2} + \frac{1}{2}x^2\sqrt{2}$	**M1 A1**
		5

Differentiation 1 MS

Q9.

5(a)	$-\sin y\frac{\mathrm{d}y}{\mathrm{d}t}=1$	M1 A1	Differentiates both sides with respect to t.
	$0<y<\pi \Rightarrow \sin y>0 \Rightarrow -\sqrt{1-\cos^2 y}\frac{\mathrm{d}y}{\mathrm{d}t}=1$	M1	Applies $\sin^2 y+\cos^2 y=1$.
	$\frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{1}{\sqrt{1-t^2}}$	A1	AG, justifies taking positive square root.
		4	
5(b)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{\sqrt{1+t^2}}$	B1	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\sqrt{\frac{1+t^2}{1-t^2}}$	B1	Finds first derivative.
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(-\sqrt{\frac{1+t^2}{1-t^2}}\right)=-\frac{t(1-t^2)^{\frac{1}{2}}(1+t^2)^{-\frac{1}{2}}+t(1+t^2)^{\frac{1}{2}}(1-t^2)^{-\frac{1}{2}}}{1-t^2}$	M1	Differentiates $\frac{\mathrm{d}y}{\mathrm{d}x}$ with respect to t.
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}}{\mathrm{d}t}\left(-\sqrt{\frac{1+t^2}{1-t^2}}\right)\times\frac{\mathrm{d}t}{\mathrm{d}x}$	M1	Applies chain rule.
	$=-\frac{t\left((1-t^2)^{\frac{1}{2}}+(1+t^2)(1-t^2)^{-\frac{1}{2}}\right)}{1-t^2}\quad\left(=-\frac{2t}{(1-t^2)^{\frac{3}{2}}}\right)$	A1	OE (simplified).
		5	

Q10.

1(a)	$\mathrm{f}'(x)=-2x\mathrm{e}^{-x^2}$	B1	Finds first derivative.
	$\mathrm{f}''(x)=4x^2\mathrm{e}^{-x^2}-2\mathrm{e}^{-x^2}$	B1	Finds second derivative.
	$\mathrm{f}(0)=1\quad \mathrm{f}'(0)=0\quad \mathrm{f}''(0)=-2$	M1	Evaluates derivatives at zero.
	$\mathrm{e}^{-x^2}=1-x^2$	M1 A1	
		5	