Differentiation 1

Q1.

The curve C has equation

$$2xy^2 + 3x^2y = 1.$$

Show that, at the point $A\ (-1,\ 1)$ on C, $\frac{dy}{dx} = -4$. [3]

Find the value of $\frac{d^2y}{dx^2}$ at A. [5]

Q2.

A curve has parametric equations

$$x = 2\sin 2t, \quad y = 3\cos 2t,$$

for $0 < t < \frac{1}{2}\pi$. For the point on the curve where $t = \frac{1}{3}\pi$, find the value of

(i) $\frac{dy}{dx}$, [3]

(ii) $\frac{d^2y}{dx^2}$. [4]

Q3.

The curve C has equation

$$xy + (x + y)^3 = 1.$$

Show that $\frac{dy}{dx} = -\frac{3}{4}$ at the point $A\ (1,\ 0)$ on C. [3]

Find the value of $\frac{d^2y}{dx^2}$ at A. [5]

Differentiation 1

Q4.

A curve has parametric equations

$$x = 2\theta - \sin 2\theta, \quad y = 1 - \cos 2\theta, \quad \text{for } -3\pi \leqslant \theta \leqslant 3\pi.$$

Show that

$$\frac{dy}{dx} = \cot\theta,$$

except for certain values of θ, which should be stated. [4]

Find the value of $\frac{d^2y}{dx^2}$ when $\theta = \frac{1}{4}\pi$. [3]

Q5.

The curve C has parametric equations

$$x = t^2, \quad y = (2 - t)^{\frac{1}{2}}, \quad \text{for } 0 \leqslant t \leqslant 2.$$

Find

(i) $\frac{d^2y}{dx^2}$ in terms of t, [5]

(ii) the mean value of y with respect to x over the interval $0 \leqslant x \leqslant 4$, [6]

(iii) the y-coordinate of the centroid of the region enclosed by C, the x-axis and the y-axis. [3]

Q6.

A curve C has equation $\cos y = x$, for $-\pi < x < \pi$.

(i) Use implicit differentiation to show that

$$\frac{d^2y}{dx^2} = -\cot y\left(\frac{dy}{dx}\right)^2. \quad [4]$$

(ii) Hence find the exact value of $\frac{d^2y}{dx^2}$ at the point $\left(\frac{1}{2}, \frac{1}{3}\pi\right)$ on C. [2]

Differentiation 1

Q7.

It is given that $y = 2^x$.

(a) By differentiating $\ln y$ with respect to x, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2^x \ln 2$. [3]

(b) Write down $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [1]

(c) Hence find the first three terms in the Maclaurin's series for 2^x. [3]

Q8.

(a) Starting from the definitions of tanh and sech in terms of exponentials, prove that

$$1 - \tanh^2\theta = \operatorname{sech}^2\theta.$$ [3]

The variables x and y are such that $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$, for $-\frac{1}{4}\pi < x < \frac{3}{4}\pi$.

(b) By differentiating the equation $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$ with respect to x, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\operatorname{cosec}\left(x + \frac{1}{4}\pi\right).$$ [4]

(c) Hence find the first three terms in the Maclaurin's series for $\tanh^{-1}\left(\cos(x + \frac{1}{4}\pi)\right)$ in the form $\frac{1}{2}\ln a + bx + cx^2$, giving the exact values of the constants a, b and c. [5]

Q9.

It is given that

$$x = \sinh^{-1} t, \quad y = \cos^{-1} t,$$

where $-1 < t < 1$.

(a) By differentiating $\cos y$ with respect to t, show that $\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{1}{\sqrt{1-t^2}}$. [4]

(b) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of t, simplifying your answer. [5]

Q10.

(a) By differentiating e^{-x^2}, find the Maclaurin's series for e^{-x^2} up to and including the term in x^2. [5]
