Differentiation 1

Q1.

The equation of a curve is $y = \frac{1}{6}(2x - 3)^3 - 4x$.

(i) Find $\frac{dy}{dx}$. [3]

(ii) Find the equation of the tangent to the curve at the point where the curve intersects the y-axis. [3]

(iii) Find the set of values of x for which $\frac{1}{6}(2x - 3)^3 - 4x$ is an increasing function of x. [3]

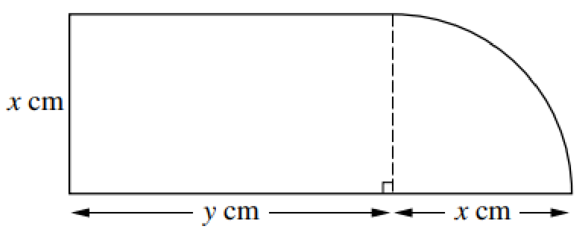
Q2.

The equation of a curve is such that $\frac{dy}{dx} = \frac{6}{\sqrt{(3x - 2)}}$. Given that the curve passes through the point $P(2, 11)$, find

(i) the equation of the normal to the curve at P, [3]

(ii) the equation of the curve. [4]

Q3.



The diagram shows a metal plate consisting of a rectangle with sides x cm and y cm and a quarter-circle of radius x cm. The perimeter of the plate is 60 cm.

(i) Express y in terms of x. [2]

(ii) Show that the area of the plate, A cm^2, is given by $A = 30x - x^2$. [2]

Given that x can vary,

(iii) find the value of x at which A is stationary, [2]

(iv) find this stationary value of A, and determine whether it is a maximum or a minimum value. [2]

Differentiation 1

Q4.

The equation of a curve is $y = 3 + 4x - x^2$.

(i) Show that the equation of the normal to the curve at the point (3, 6) is $2y = x + 9$. [4]

(ii) Given that the normal meets the coordinate axes at points A and B, find the coordinates of the mid-point of AB. [2]

(iii) Find the coordinates of the point at which the normal meets the curve again. [4]

Q5.

The equation of a curve is $y = \dfrac{9}{2 - x}$.

(i) Find an expression for $\dfrac{dy}{dx}$ and determine, with a reason, whether the curve has any stationary points. [3]

Q6.

The length, x metres, of a Green Anaconda snake which is t years old is given approximately by the formula

$$x = 0.7\sqrt{(2t - 1)},$$

where $1 \leqslant t \leqslant 10$. Using this formula, find

(i) $\dfrac{dx}{dt}$, [2]

(ii) the rate of growth of a Green Anaconda snake which is 5 years old. [2]

Q7.

A curve has equation $y = \dfrac{1}{x - 3} + x$.

(i) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [2]

(ii) Find the coordinates of the maximum point A and the minimum point B on the curve. [5]

Differentiation 1

Q8.

A curve has equation $y = f(x)$. It is given that $f'(x) = 3x^2 + 2x - 5$.

(i) Find the set of values of x for which f is an increasing function. [3]

Q9.

The volume of a spherical balloon is increasing at a constant rate of $50\,\text{cm}^3$ per second. Find the rate of increase of the radius when the radius is $10\,\text{cm}$. [Volume of a sphere $= \frac{4}{3}\pi r^3$.] [4]

Q10.

A curve has equation $y = \dfrac{4}{3x - 4}$ and $P\,(2, 2)$ is a point on the curve.

(i) Find the equation of the tangent to the curve at P. [4]

(ii) Find the angle that this tangent makes with the x-axis. [2]

Q11.

A curve is such that $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{x}} - 1$ and $P\,(9, 5)$ is a point on the curve.

(ii) Find the coordinates of the stationary point on the curve. [3]

(iii) Find an expression for $\dfrac{d^2y}{dx^2}$ and determine the nature of the stationary point. [2]

(iv) The normal to the curve at P makes an angle of $\tan^{-1} k$ with the positive x-axis. Find the value of k. [2]

Q12.

A curve has equation $y = 3x^3 - 6x^2 + 4x + 2$. Show that the gradient of the curve is never negative. [3]
