Differentiation 2

Q1.

A curve C has equation $x^2 + 4xy - y^2 + 20 = 0$. Show that, at stationary points on C, $x = -2y$. [3]

Find the coordinates of the stationary points on C, and determine their nature by considering the value of $\frac{d^2y}{dx^2}$ at the stationary points. [8]

Q2.

A curve C has equation $\tan y = x$, for $x > 0$.

(i) Use implicit differentiation to show that

$$\frac{d^2y}{dx^2} = -2x\left(\frac{dy}{dx}\right)^2.$$ [3]

(ii) Hence find the value of $\frac{d^2y}{dx^2}$ at the point $(1, \frac{1}{4}\pi)$ on C. [2]

Q3.

A curve C has equation $x^3 - 3xy + y^2 = 4$. Find the value of $\frac{d^2y}{dx^2}$ at the point $(0, 2)$ of C. [7]

Q4.

The curve C has equation $2x^3 + 3x^2y - 3y^3 - 16 = 0$.

(i) Find the coordinates of the point A on C at which $\frac{dy}{dx} = 0$ and $x \neq 0$. [5]

(ii) Find the value of $\frac{d^2y}{dx^2}$ at A. [3]

Q5.

Find the Maclaurin's series for $\tan\left(x + \frac{1}{4}\pi\right)$ up to and including the term in x^2. [5]

Differentiation 2

Q6.

The curve C has equation

$$y^2 + (xy+1)^2 = 5.$$

(a) Show that, at the point $(1, 1)$ on C, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{3}$. [3]

(b) Find the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at the point $(1, 1)$. [5]

Q7.

(a) It is given that $y = \operatorname{sech}^{-1}\left(x + \frac{1}{2}\right)$.

Express $\cosh y$ in terms of x and hence show that $\sinh y \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\left(x + \frac{1}{2}\right)^2}$. [3]

(b) Find the first three terms in the Maclaurin's series for $\operatorname{sech}^{-1}\left(x + \frac{1}{2}\right)$ in the form

$$\ln a + bx + cx^2,$$

where a, b and c are constants to be determined. [7]

Q8.

The curve C has parametric equations

$$x = 2\cosh t, \quad y = \tfrac{3}{2}t - \tfrac{1}{4}\sinh 2t, \quad \text{for } 0 \leqslant t \leqslant 1.$$

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}t}$ and show that $\frac{\mathrm{d}y}{\mathrm{d}t} = 1 - \sinh^2 t$. [3]

Q9.

Find the Maclaurin's series for $\ln\cosh x$ up to and including the term in x^4. [7]

Q10.

Find the Maclaurin's series for $\mathrm{e}^x \tan x$ from first principles up to and including the term in x^2. [5]
