Differentiation 2 MS

Q1.

8	$2x+4(xy'+y)-2yy'=0$ (*) $y'=0 \Rightarrow 2x+4y=0 \Rightarrow x=-2y$ (AG)	M1A1 A1	[3]
	At stationary points $4y^2-8y^2-y^2+20=0 \Rightarrow 5y^2=20 \Rightarrow y=\pm 2$ Coordinates of stationary points are (4 , –2) and (–4,2)	M1A1 A1	
	From (*): $x+2(xy'+y)-yy'=0$ Differentiating: $1+2(xy''+y'+y')-\left(yy''+[y']^2\right)=0$ (or by quotient rule)	M1A1	
	At (4 , –2) with $y'=0$: $1+8y''+2y''=0 \Rightarrow y''=-\frac{1}{10} \Rightarrow$ maximum.	M1A1	
	At (–4 , 2) with $y'=0$: $1-8y''-2y''=0 \Rightarrow y''=\frac{1}{10} \Rightarrow$ minimum.	A1	[8]

Q2.

3(i)	$(1+x^2)\frac{dy}{dx}=1$ **or** $\sec^2 y\frac{dy}{dx}=1 \Rightarrow (1+x^2)\frac{dy}{dx}=1$ **or** $\frac{dy}{dx}=\cos^2 y$	M1	Using implicit differentiation
	$\Rightarrow 2x\frac{dy}{dx}+(1+x^2)\frac{d^2y}{dx^2}=0 \Rightarrow \frac{d^2y}{dx^2}=-2x\left(\frac{dy}{dx}\right)^2$ (AG)	M1 A1	M1 for good attempt at product rule
	Alt method: $\frac{dy}{dx}=\cos^2 y \Rightarrow \frac{d^2y}{dx^2}=\cos y(-\sin y)\frac{dy}{dx}=-2x\left(\frac{dy}{dx}\right)^2$	M1 A1	M1 for good attempt at implicit differentiation
	Total:	**3**	
3(ii)	$y'(1)=\cos^2\left(\frac{\pi}{4}\right) \Rightarrow y'(1)=\frac{1}{2}$	B1	
	$\Rightarrow y''(1)=-\frac{1}{2}$	B1 FT	
	Total:	**2**	

Q3.

4	$3x^2-\left(3y+3x\frac{dy}{dx}\right)+2y\frac{dy}{dx}=0$	B1B1	SOI
	At (0,2): $-6+4\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{3}{2}$	B1	
	$6x-3\frac{dy}{dx}-\left(3\frac{dy}{dx}+3x\frac{d^2y}{dx^2}\right)$	M1A1	
	$+2\left(\frac{dy}{dx}\right)^2+2y\frac{d^2y}{dx^2}=0$	A1	
	At (0,2): $-\frac{9}{2}-\frac{9}{2}+\frac{9}{2}+4\frac{d^2y}{dx^2}=0 \Rightarrow \frac{d^2y}{dx^2}=\frac{9}{8}$	A1	
	Total:	**7**	

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Q4.

5(i)	$6x^2+6xy+3x^2y'-9y^2y'=0\ (*)\ \Rightarrow 2x(x+y)=(3y^2-x^2)y'$	M1A1	
	$y'=0 \text{ and } x\neq 0 \Rightarrow x=-y$	M1A1	
	$\Rightarrow 2x^3-3x^3+3x^3=16 \Rightarrow A$ is $(2,-2)$	A1	
		5	
5(ii)	$12x+6xy'+6y+6xy'+3x^2y''-\left[18y(y')^2+9y^2y''\right]=0$	*M1	
	$x=2\quad y=-2\quad y'=0\ \Rightarrow 8-4+4y''-12y''=0$	DM1	
	$\Rightarrow y''=\frac{1}{2}$	A1	
		3	

Q5.

1	$f'(x)=\sec^2\left(x+\frac{1}{4}\pi\right)$	B1	Finds first derivative.
	$f''(x)=2\sec^2\left(x+\frac{1}{4}\pi\right)\tan\left(x+\frac{1}{4}\pi\right)$	B1	Finds second derivative.
	$f(0)=1\quad f'(0)=2\quad f''(0)=4$	M1	Evaluates derivatives at zero.
	$\tan\left(x+\frac{1}{4}\pi\right)=1+2x+2x^2$	M1 A1	
		5	

Q6.

5(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(y^2\right)=2yy'$	B1	Differentiates y^2 correctly.
	$\frac{\mathrm{d}}{\mathrm{d}x}\left((xy+1)^2\right)=2(xy+1)(xy'+y)$	B1	Differentiates $(xy+1)^2$ correctly.
	$(1)y'+((1)+1)(y'+1)=0 \Rightarrow y'=-\frac{2}{3}$	B1	Substitutes (1,1), AG. $y'=-\frac{xy^2+y}{y+x^2y+x}$
		3	
5(b)	$yy''+(y')^2$	B1	Differentiates yy'.
	$+(xy+1)(xy''+y'+y')+(xy'+y)(xy'+y)=0$	B1 B1	Differentiates $(xy+1)(xy'+y)$.
	$y''+\left(-\frac{2}{3}\right)^2+2\left(y''-\frac{4}{3}\right)+\left(-\frac{2}{3}+1\right)\left(\frac{1}{3}\right)=0$	M1	Substitutes (1,1) and $y'=-\frac{2}{3}$
	$y''=\frac{19}{27}$	A1	
		5	

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5(b)	Alternative $y'' = -\frac{(y+x^2y+x)(2xyy'+y^2+y') - (xy^2+y)(y'+2xy+x^2y'+1)}{(y+x^2y+x)^2}$	M1 A1 A1	Differentiate $y' = -\frac{xy^2+y}{y+x^2y+x}$ using quotient rule. A1 for differentiating xy^2 correctly. A1 for everything correct.
	$y'' = \frac{19}{27}$	M1 A1	Substitutes $(1,1)$ and $y' = -\frac{2}{3}$.
		5	

Q7.

7(a)	$\text{sech}\, y = \frac{1}{\cosh y} = x + \frac{1}{2} \Rightarrow \cosh y = \left(x+\frac{1}{2}\right)^{-1}$	B1	Relates to $\cosh y$,
	$\sinh y \frac{dy}{dx}$	B1	Differentiates LHS.
	$-\left(x+\frac{1}{2}\right)^{-2}$	B1	Differentiates RHS. AG.
		3	
7(b)	$\sinh \frac{d^2y}{dx^2} + \cosh y\left(\frac{dy}{dx}\right)^2 = 2\left(x+\frac{1}{2}\right)^{-3}$	M1 A1	M1 A1 for LHS. B1 for RHS.
		B1	
	$y(0) = \text{sech}^{-1}\left(\frac{1}{2}\right) = \cosh^{-1}(2) = \ln\left(2+\sqrt{3}\right)$	M1 A1	Relates to $\cosh^{-1}$ and uses logarithmic form.
	$y'(0) = -\frac{4}{\sqrt{3}} \quad y''(0) = \frac{16}{3\sqrt{3}}$	M1	Evaluates derivatives at $x = 0$.
	$y = \ln\left(2+\sqrt{3}\right) - \frac{4}{\sqrt{3}}x + \frac{8}{3\sqrt{3}}x^2$	A1	
	Alternative method for question 7(b)		
	$\frac{dy}{dx} = -\frac{1}{\left(x+\frac{1}{2}\right)^2\sqrt{\left(x+\frac{1}{2}\right)^{-2}-1}} = -\frac{1}{\left(x+\frac{1}{2}\right)\sqrt{\frac{3}{4}-x-x^2}}$	B1	
	$\frac{d^2y}{dx^2} = \frac{1}{2}\left(x+\frac{1}{2}\right)^{-1}\left(\frac{3}{4}-x-x^2\right)^{-\frac{3}{2}}(-1-2x) + \left(\frac{3}{4}-x-x^2\right)^{-\frac{1}{2}}\left(x+\frac{1}{2}\right)^{-2}$	M1 A1	
	$y(0) = \text{sech}^{-1}\left(\frac{1}{2}\right) = \cosh^{-1}(2) = \ln\left(2+\sqrt{3}\right)$	M1 A1	Relates to $\cosh^{-1}$ and uses logarithmic form.
7(b)	$y'(0) = -\frac{4}{\sqrt{3}} \quad y''(0) = \frac{16}{3\sqrt{3}}$	M1	Evaluates derivatives at $x = 0$.
	$y = \ln\left(2+\sqrt{3}\right) - \frac{4}{\sqrt{3}}x + \frac{8}{3\sqrt{3}}x^2$	A1	
		7	

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Q8.

8(a)	$\frac{dx}{dt} = 2\sinh t$	B1	
	$\frac{dy}{dt} = \frac{3}{2} - \frac{1}{2}\cosh 2t = 1 - \left(\frac{1}{2}\cosh 2t - \frac{1}{2}\right) = 1 - \sinh^2 t$	M1 A1	Applies $2\sinh^2 t = \cosh 2t - 1$, AG.
		3	

Q9.

2	$\frac{dy}{dx} = \tanh x$	B1	Finds first derivative.
	$\frac{d^2y}{dx^2} = \text{sech}^2 x$	B1	Finds second derivative.
	$\frac{d^3y}{dx^3} = -2\tanh x\,\text{sech}^2 x = -2\frac{\sinh x}{\cosh^3 x}$	M1 A1	Finds third derivative.
	$\frac{d^4y}{dx^4} = -2\frac{\cosh^2(x) - 3\sinh^2(x)}{\cosh^4(x)} = -2\,\text{sech}^4 x + 4\tanh^2 x\,\text{sech}^2 x$	B1	Finds fourth derivative. Alternative: $6\tanh^2 x\,\text{sech}^2 x - 2\,\text{sech}^2 x$.
	$y(0) = y^{(1)}(0) = y^{(3)}(0) = 0 \quad y^{(2)}(0) = 1 \quad y^{(4)}(0) = -2$	M1	Evaluates derivatives at $x = 0$.
	$y = \frac{1}{2}x^2 - \frac{1}{12}x^4$	A1	CWO
		7	

Q10.

1	$e^x\left(\tan x + \sec^2 x\right)$	B1	Finds first derivative.
	$e^x\left(\tan x + 2\sec^2 x + 2\sec^2 x \tan x\right)$	M1 A1	Finds second derivative.
	$y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 2$	M1	Evaluates derivatives at $x = 0$.
	$y = x + x^2$	A1	
		5	