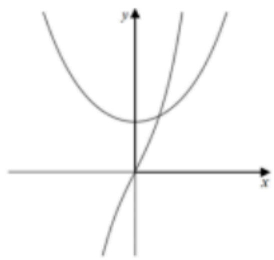
Hyperbolic Functions 1 MS

Q1.

5(a)	$\cosh a = 2\sinh a \cosh a \Rightarrow \sinh a = \frac{1}{2}$	M1 A1
	$a = \sinh^{-1}\frac{1}{2} = \ln\left(\frac{1}{2}+\sqrt{\frac{1}{4}+1}\right)$	M1
	$a = \ln\left(\frac{1}{2}+\frac{1}{2}\sqrt{5}\right)$	A1
		4

5(b)	 (**B1** for C_1 correct, **B1** for C_2 correct and intersecting C_1 in the first quadrant)	B1 B1
		2
5(c)	$\int_0^a \sqrt{1+\sinh^2 x}\,dx$	M1
	$\int_0^a \sqrt{\cosh^2 x}\,dx = \int_0^a \cosh x\,dx$	M1 A1
	$[\sinh x]_0^a = \sinh a$	M1
	$\frac{1}{2}$	A1
		5

Q2.

6(a)	$\tanh\theta = \frac{e^{\theta}-e^{-\theta}}{e^{\theta}+e^{-\theta}} \quad \operatorname{sech}\theta = \frac{2}{e^{\theta}+e^{-\theta}}$	B1
	$1-\left(\frac{e^{\theta}-e^{-\theta}}{e^{\theta}+e^{-\theta}}\right)^2 = \frac{\left(e^{\theta}+e^{-\theta}\right)^2-\left(e^{\theta}-e^{-\theta}\right)^2}{\left(e^{\theta}+e^{-\theta}\right)^2} = \frac{4}{\left(e^{\theta}+e^{-\theta}\right)^2}$	M1
	$= \operatorname{sech}^2\theta$	A1
		3

Q3.

6(a)	$\sinh x = \frac{1}{2}\left(e^{x}-e^{-x}\right) \quad \cosh x = \frac{1}{2}\left(e^{x}+e^{-x}\right)$	B1	Writes in exponential from.
	$\frac{1}{2}\left(e^{x}-e^{-x}\right)^2 = \frac{1}{2}\left(e^{2x}-2+e^{-2x}\right) = \frac{1}{2}\left(e^{2x}+e^{-2x}\right)-1$	M1	Expands.
	$\cosh 2x - 1$	A1	AG.
		3	

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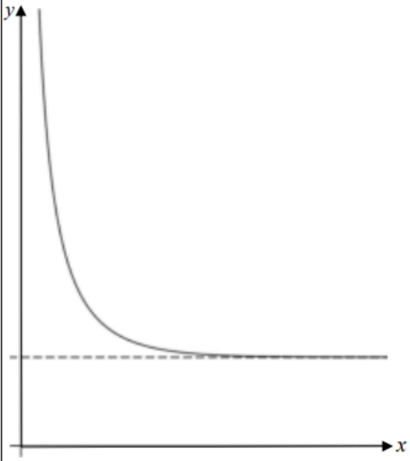
Q4.

8(a)		**B1**	Correct shape and position, not too truncated.
	$x=0,\quad y=1$	**B1**	States equations of asymptotes.
		2	
8(b)	$\coth x=\dfrac{e^x+e^{-x}}{e^x-e^{-x}}\quad \operatorname{cosech} x=\dfrac{2}{e^x-e^{-x}}$	**B1**	
	$\left(\dfrac{e^x+e^{-x}}{e^x-e^{-x}}\right)^2-\dfrac{4}{\left(e^x-e^{-x}\right)^2}=\dfrac{e^{2x}+e^{-2x}-2}{\left(e^x-e^{-x}\right)^2}=1$	**M1 A1**	Writes over common denominator, AG.
		3	
8(c)	$\dfrac{dy}{dx}=-\dfrac{\operatorname{sech}^2\left(\frac{1}{2}x\right)}{2\tanh\left(\frac{1}{2}x\right)}=-\dfrac{1}{2\sinh\left(\frac{1}{2}x\right)\cosh\left(\frac{1}{2}x\right)}$ Or $\dfrac{dy}{dx}=-\dfrac{\operatorname{cosech}^2\left(\frac{1}{2}x\right)}{2\coth\left(\frac{1}{2}x\right)}=-\dfrac{1}{2\sinh\left(\frac{1}{2}x\right)\cosh\left(\frac{1}{2}x\right)}$	**M1 A1**	Uses chain rule.
	$=-\dfrac{1}{\sinh(x)}=-\operatorname{cosech} x$	**A1**	AG
		3	
8(d)	$\int_a^{2a}\sqrt{1+\operatorname{cosech}^2 x}\,dx$	**M1**	Forms correct integral.
	$\int_a^{2a}\sqrt{\coth^2 x}\,dx=\int_a^{2a}\coth x\,dx$	**M1 A1**	Uses $\coth^2 x-\operatorname{cosech}^2 x=1$.
	$=\left[\ln\sinh x\right]_a^{2a}=\ln\sinh 2a-\ln\sinh a$	**M1**	Integrates and substitutes limits.
	$=\ln\dfrac{\sinh 2a}{\sinh a}=\ln\left(2\cosh a\right)$	**M1**	Combines logarithms and uses double angle formula.
	$\ln\left(2\cosh a\right)=\ln 4\Rightarrow\cosh a=2$	**A1**	AG
	$a=\ln\left(2+\sqrt{2^2-1}\right)=\ln\left(2+\sqrt{3}\right)$	**A1**	Must reject $\ln\left(2-\sqrt{3}\right)$.
		7	

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Q5.

8(a)	$\cosh A = \frac{1}{2}\left(e^{A} + e^{-A}\right)$	**B1**	Writes in exponential form
	$2\cosh^2 A = \frac{1}{2}\left(e^{2A} + 2 + e^{-2A}\right) = \frac{1}{2}\left(e^{2A} + e^{-2A}\right) + 1 = \cosh 2A + 1$	**M1 A1**	Expands, AG.
		3	

Q6.

8(a)	$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \quad \operatorname{sech} x = \frac{2}{e^{x} + e^{-x}}$	**B1**	Writes in exponential form.
	$1 - \left(\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\right)^2 = \frac{\left(e^{x} + e^{-x}\right)^2 - \left(e^{x} - e^{-x}\right)^2}{\left(e^{x} + e^{-x}\right)^2} = \frac{4}{\left(e^{x} + e^{-x}\right)^2}$	**M1**	Writes over common denominator.
	$\operatorname{sech}^2 x$	**A1**	AG
		3	