Hyperbolic Functions 1

Q1.

The curves $C_1 : y = \cosh x$ and $C_2 : y = \sinh 2x$ intersect at the point where $x = a$.

(a) Find the exact value of a, giving your answer in logarithmic form. [4]

(b) Sketch C_1 and C_2 on the same diagram. [2]

(c) Find the exact value of the length of the arc of C_1 from $x = 0$ to $x = a$. [5]

Q2.

(a) Starting from the definitions of tanh and sech in terms of exponentials, prove that

$$1 - \tanh^2\theta = \operatorname{sech}^2\theta.$$ [3]

Q3.

(a) Starting from the definitions of sinh and cosh in terms of exponentials, prove that

$$2\sinh^2 x = \cosh 2x - 1.$$ [3]

Q4.

(a) Sketch the graph of $y = \coth x$ for $x > 0$ and state the equations of the asymptotes. [2]

(b) Starting from the definitions of coth and cosech in terms of exponentials, prove that

$$\coth^2 x - \operatorname{cosech}^2 x = 1.$$ [3]

The curve C has equation $y = \ln\coth\left(\frac{1}{2}x\right)$ for $x > 0$.

(c) Show that $\frac{dy}{dx} = -\operatorname{cosech} x$. [3]

(d) It is given that the arc length of C from $x = a$ to $x = 2a$ is $\ln 4$, where a is a positive constant.

Show that $\cosh a = 2$ and find, in logarithmic form, the exact value of a. [7]

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Q5.

(a) Starting from the definition of cosh in terms of exponentials, prove that

$$2\cosh^2 A = \cosh 2A + 1.$$ [3]

Q6.

(a) Starting from the definitions of tanh and sech in terms of exponentials, prove that

$$1 - \tanh^2 x = \operatorname{sech}^2 x.$$ [3]
