Hyperbolic Functions 2 MS

Q1.

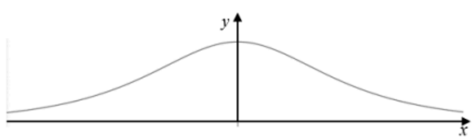
2(a)	$\cosh x=\frac{1}{2}\left(e^{x}+e^{-x}\right)\quad \sinh x=\frac{1}{2}\left(e^{x}-e^{-x}\right)$	**B1**	
	$\frac{1}{2}\left(e^{x}-e^{-x}\right)^{2}+1=\frac{1}{2}\left(e^{2x}-e^{-2x}\right)=\cosh 2x$	**M1 A1**	Expands, AG.
		3	
2(b)	$2\sinh^{2}x-k\sinh x+1=0$	**M1 A1**	Applies identity.
	$k^{2}-8>0$	**M1 A1**	Sets discriminant positive.
	$k<-\sqrt{8},\ k>\sqrt{8}$	**A1**	
		5	

Q2.

4(a)	$\cosh x=\frac{1}{2}\left(e^{x}+e^{-x}\right)\quad \sinh x=\frac{1}{2}\left(e^{x}-e^{-x}\right)$	**B1**	
	$\frac{1}{4}\left(e^{2x}+2+e^{-2x}-e^{2x}+2-e^{-2x}\right)=1$	**M1 A1**	Expands, AG. Clear LHS to RHS for A1.
		3	

Q3.

4(a)	$\cosh x=\frac{1}{2}\left(e^{x}+e^{-x}\right)\quad \sinh x=\frac{1}{2}\left(e^{x}-e^{-x}\right)$	**B1**	
	$\frac{1}{4}\left(e^{2x}+2+e^{-2x}-e^{2x}+2-e^{-2x}\right)=1$	**M1 A1**	Expands, AG. Clear LHS to RHS for A1.
		3	

4(c)		**B1**	Correct shape, symmetrical about $x=0$.
	$y=0$	**B1**	Accept labels on their sketch.
		2	

Q4.

8(a)	$\operatorname{sech} t=\dfrac{2}{e^{t}+e^{-t}}\quad \tanh t=\dfrac{e^{t}-e^{-t}}{e^{t}+e^{-t}}$	**B1**	
	$1-\left(\dfrac{2}{e^{t}+e^{-t}}\right)^{2}=\dfrac{\left(e^{t}+e^{-t}\right)^{2}-4}{\left(e^{t}+e^{-t}\right)^{2}}=\dfrac{e^{2t}+2+e^{-2t}-4}{\left(e^{t}+e^{-t}\right)^{2}}=\dfrac{\left(e^{t}-e^{-t}\right)^{2}}{\left(e^{t}+e^{-t}\right)^{2}}$	**M1 A1**	Expands, gets to $\dfrac{e^{2t}+2+e^{-2t}-4}{\left(e^{t}+e^{-t}\right)^{2}}$ for M1, AG.
		3	

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Q5.

5(a)	$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$	B1	
	$\frac{2}{4}\left(e^x + e^{-x}\right)^2 = \frac{1}{2}\left(e^{2x} + e^{-2x} + 2\right) = \cosh 2x + 1$	M1 A1	Expands, AG.
		3	

Q6.

6(a)	$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right) \quad \sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$	B1	
	$\frac{1}{2}\left(e^x - e^{-x}\right)\left(e^x + e^{-x}\right) = \frac{1}{2}\left(e^{2x} - e^{-2x}\right) = \sinh 2x$	M1 A1	Expands, AG.
		3	

Q7.

7(a)	$\cosh 2A = \frac{1}{2}\left(e^{2A} + e^{-2A}\right) \quad \sinh A = \frac{1}{2}\left(e^A - e^{-A}\right)$	B1	
	$2\sinh^2 A = \frac{1}{2}\left(e^A - e^{-A}\right)^2 = \frac{1}{2}\left(e^{2A} - 2 + e^{-2A}\right) = \cosh 2A - 1$	M1 A1	Expands, AG. A0 for mixing variables e.g. $\sinh A = \frac{1}{2}\left(e^x - e^{-x}\right)$.
		3	