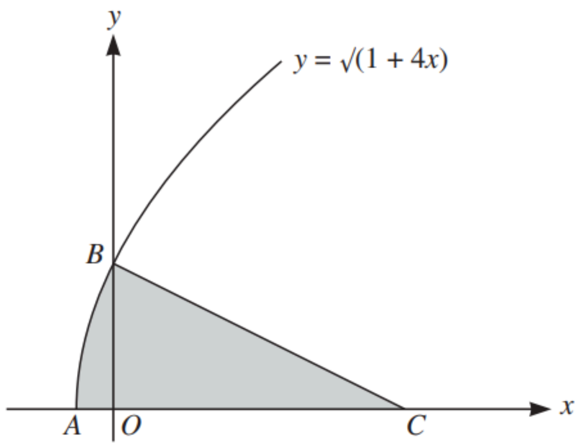
Integration 2

Q1.

A curve is such that $\frac{dy}{dx} = \frac{6}{x^2}$ and (2, 9) is a point on the curve. Find the equation of the curve. [3]

Q2.



The diagram shows the curve $y = \sqrt{(1 + 4x)}$, which intersects the x-axis at A and the y-axis at B. The normal to the curve at B meets the x-axis at C. Find

(i) the equation of BC, [5]

(ii) the area of the shaded region. [5]

Q3.

A curve is such that $\frac{dy}{dx} = \sqrt{(2x + 5)}$ and (2, 5) is a point on the curve. Find the equation of the curve. [4]

Q4.

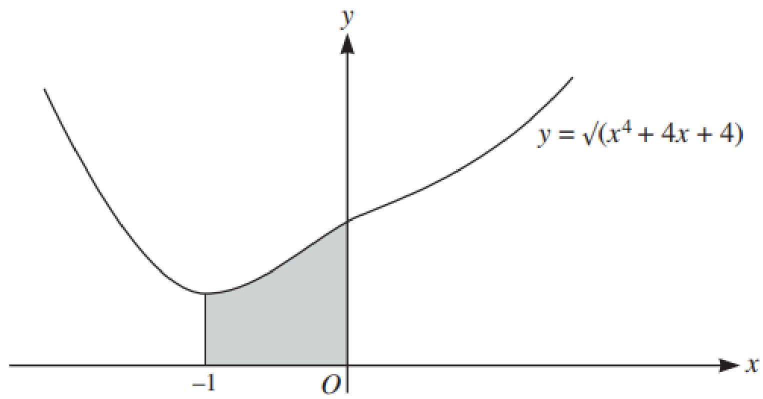
The equation of a curve is $y = \frac{2}{\sqrt{(5x - 6)}}$.

(i) Find the gradient of the curve at the point where $x = 2$. [3]

(ii) Find $\int \frac{2}{\sqrt{(5x - 6)}}\,dx$ and hence evaluate $\int_2^3 \frac{2}{\sqrt{(5x - 6)}}\,dx$. [4]

Integration 2

Q5.



The diagram shows the curve $y = \sqrt{(x^4 + 4x + 4)}$.

(i) Find the equation of the tangent to the curve at the point (0, 2). [4]

(ii) Show that the x-coordinates of the points of intersection of the line $y = x + 2$ and the curve are given by the equation $(x + 2)^2 = x^4 + 4x + 4$. Hence find these x-coordinates. [4]

(iii) The region shaded in the diagram is rotated through 360° about the x-axis. Find the volume of revolution. [4]

Q6.

A line has equation $y = 2x + c$ and a curve has equation $y = 8 - 2x - x^2$.

(i) For the case where the line is a tangent to the curve, find the value of the constant c. [3]

(ii) For the case where $c = 11$, find the x-coordinates of the points of intersection of the line and the curve. Find also, by integration, the area of the region between the line and the curve. [7]

Q7.

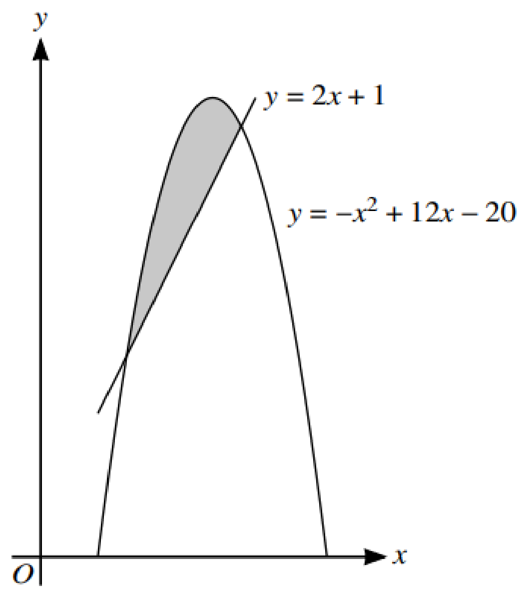
A curve is such that $\frac{dy}{dx} = \frac{12}{\sqrt{(4x + a)}}$, where a is a constant. The point $P\,(2, 14)$ lies on the curve and the normal to the curve at P is $3y + x = 5$.

(i) Show that $a = 8$. [3]

(ii) Find the equation of the curve. [4]

Integration 2

Q8.



The diagram shows the curve $y = -x^2 + 12x - 20$ and the line $y = 2x + 1$. Find, showing all necessary working, the area of the shaded region. [8]

Q9.

The function f is defined for $x > 0$ and is such that $f'(x) = 2x - \frac{2}{x^2}$. The curve $y = f(x)$ passes through the point $P(2, 6)$.

(i) Find the equation of the normal to the curve at P. [3]

(ii) Find the equation of the curve. [4]

(iii) Find the x-coordinate of the stationary point and state with a reason whether this point is a maximum or a minimum. [4]
