Integration 2 MS

Q1.

1	$\frac{dy}{dx}=\frac{6}{x^2}$ $y=-6x^{-1}+c$ Uses (2, 9) → $c = 12$ $y=-6x^{-1}+12$	B1 M1 A1 [3]	Integration only – unsimplified Uses (2, 9) in an integral

Q2.

11	$y=\sqrt{1+4x}$ (i) $\frac{dy}{dx}=\frac{1}{2}(1+4x)^{-\frac{1}{2}}\times 4$ $= 2$ at B (0, 1) Gradient of normal = $-\frac{1}{2}$ Equation $y-1=-\frac{1}{2}x$	B1 B1 M1 M1 A1 [5]	B1 Without "×4". B1 for "×4" even if first B mark lost. Use of $m_1m_2=-1$ Correct method for eqn.
	(ii) At A $x=-\frac{1}{4}$ $\int\sqrt{1+4x}\,dx=\frac{(1+4x)^{\frac{3}{2}}}{\frac{3}{2}}\div 4$ Limits $-\frac{1}{4}$ to 0 → $\frac{1}{6}$ Area $BOC=\frac{1}{2}\times 2\times 1=1$ → Shaded area = $\frac{7}{6}$	B1 B1 B1 B1 B1√ [5]	 B1 Without the "÷4". For "÷4" even if first B mark lost. For 1 + his "1/6".

Q3.

1	$\frac{dy}{dx}=\sqrt{2x+5}$ $\frac{(2x+5)^{\frac{3}{2}}}{\frac{3}{2}}\div 2 \quad (+c)$ Uses (2, 5) → $c=-4$	B1 B1 M1 A1	[4]	B1 Everything without "÷2". B1 "÷2" Uses point in an integral.

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Q4.

3 $y = \dfrac{2}{\sqrt{5x-6}}$		
(i) $\dfrac{dy}{dx} = 2 \times -½ \times (5x-6)^{-\frac{3}{2}} \times 5$ $\rightarrow -\frac{5}{8}$	B1 B1 B1 [3]	B1 without '×5'. B1 For '×5' Use of '*uv*' or '*u/v*' ok.
(ii) integral $= \dfrac{2\sqrt{5x-6}}{\frac{1}{2}} \div 5$	B1 B1	B1 without '÷5'. B1 for '÷ 5'
Uses 2 to 3 $\rightarrow 2.4 - 1.6 = 0.8$	M1 A1 [4]	Use of limits in an integral.

Q5.

9 $\dfrac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$	M1A1	Attempt differentiation & set to zero
$x + 2 = \pm k$	DM1	Attempt to solve
$x = -2 \pm k$	A1	cao
$\dfrac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$	M1	Attempt to differentiate again
	M1	Sub their x value with k in it into $\dfrac{d^2y}{dx^2}$
When $x = -2 = k$, $\dfrac{d^2y}{dx^2} = \left(\dfrac{2}{k}\right)$ which is (> 0) min	A1	Only 1 of bracketed items needed for each
When $x = -2 - k$, $\dfrac{d^2y}{dx^2} = \left(\dfrac{2}{-k}\right)$ which is (< 0) max	A1 [8]	but $\dfrac{d^2y}{dx^2}$ and x need to be correct.

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Q6.

11 **(i)** $x^2 + 4x + c - 8\ (= 0)$ $16 - 4(c - 8) = 0$ $c = 12$	**M1** **M1** **A1**	Attempt to simplify to 3-term quadratic Apply $b^2 - 4ac = 0$. '= 0' soi
OR $-2 - 2x = 2 \rightarrow x = (-2)$ $-4 + c = 8 + 4 - 4$ $c = 12$	**M1** **M1** **A1** **[3]**	Equate derivs of curve and line. Expect $x = -2$ Sub *their* $x = -2$ into line and curve, and equate
(ii) $x^2 + 4x + 3 \rightarrow (x + 1)(x + 3)\ (= 0) \rightarrow$ $x = -1$ or -3	**B1**	
$\int(8 - 2x - x^2) - [\int(2x + 11)$ or area of trapezium$]$	**M1M1**	Attempt to integrate. At some stage subtract
$\left[8x - x^2 - \frac{x^3}{3}\right] - \left[x^2 + 11x\right]$ or $\left[8x - x^2 - \frac{x^3}{3}\right] - \frac{1}{2}(5 + 9) \times 2$	**A1B1**	A1 for curve, B1 for line **OR** $\left[-3x - 2x^2 - \frac{x^3}{3}\right]$ A2,1,0
Apply *their* limits to at least integral for curve $1\frac{1}{3}$ oe	**M1** **A1** **[7]**	For M marks allow reversed limits and/or subtraction of areas but then final A0

Q7.

6 $\frac{dy}{dx} = \frac{12}{\sqrt{4x + a}}$ $P\ (2, 14)$ Normal $3y + x = 44$		
(i) m of normal $= -\frac{1}{3}$	B1	co
$\frac{dy}{dx} = 3 = \frac{12}{\sqrt{4x + a}} \rightarrow a = 8$	M1 A1 [3]	Use of $m_1 m_2 = -1$. AG.
(ii) $\int y = 12(4x + a)^{\frac{1}{2}} \div \frac{1}{2}$ $\div 4$ $(+c)$	B1 B1	Correct without "÷4". for "÷4".
Uses (2, 14) $c = -10$	M1 A1 [4]	Uses in an integral only. Dep 'c'. co All 4 marks can be given in (i)

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Q8.

10 pts of intersection $2x + 1 = -x^2 + 12x - 20$ $\rightarrow x = 3, 7$	M1A1	Attempt at soln of sim eqns. co
Area of trapezium = $\frac{1}{2}(4)(7 + 15) = 44$ (or $\int (2x+1)\,dx$ from 3 to 7 = 44)	M1A1	Either method ok. co
Area under curve = $-\frac{1}{3}x^3 + 6x^2 - 20x$	B2,1	−1 each term incorrect
Uses 3 to 7 $\rightarrow$ $(54\frac{2}{3})$	DM1	Correct use of limits (Dep 1st M1)
Shaded area = $10\frac{2}{3}$	A1 [8]	co
OR		
$\int_3^7 \left(-x^2 + 10x - 21\right) = -\frac{x^3}{3} + 5x^2 - 21x$		Functions subtracted before integration
M1 subtraction, A1A1A1 for integrated terms, DM1 correct use of limits, A1		Subtraction reversed allow A3A0. Limits reversed allow DM1A0

Q9.

11	**(i)**	For $y = (4x+1)^{\frac{1}{2}}$, $\frac{dy}{dx} = \left[\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right] \times [4]$	**B1B1**	
		When $x = 2$, gradient $m_1 = \frac{2}{3}$	**B1**√	Ft from *their* derivative above
		For $y = \frac{1}{2}x^2 + 1$, $\frac{dy}{dx} = x \rightarrow$ gradient $m_2 = 2$	**B1**	
		$\alpha = \tan^{-1} m_2 - \tan^{-1} m_1$	**M1**	
		$\alpha = 63.43 - 33.69 = 29.7$ cao	**A1** **[6]**	
	(ii)	$\int (4x+1)^{\frac{1}{2}}\,dx = \left[\frac{(4x+1)^{\frac{3}{2}}}{2/3}\right] \div [4]$	**B1B1**	
		$\int \left(\frac{1}{2}x^2 + 1\right)dx = \frac{1}{6}x^3 + x$	**B1**	
		$\int_0^2 (4x+1)^{\frac{1}{2}}\,dx = \frac{1}{6}[27-1]$, $\int_0^2 \left(\frac{1}{2}x^2 + 1\right)dx = \left[\frac{8}{6} + 2\right]$	**M1**	Apply limits $0 \rightarrow 2$ to at least the 1st integral
		$\frac{13}{3} - \frac{10}{3}$	**M1**	Subtract the integrals (at some stage)
		1	**A1** **[6]**	