Matrices 1

Q1.

A 3×3 matrix $\mathbf{A}$ has eigenvalues -1, 1, 2, with corresponding eigenvectors

$$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix},$$

respectively. Find

(i) the matrix $\mathbf{A}$,

(ii) $\mathbf{A}^{2n}$, where n is a positive integer.

[14]

Q2.

A matrix $\mathbf{A}$ has eigenvalues -1, 1 and 2, with corresponding eigenvectors

$$\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix},$$

respectively. Find $\mathbf{A}$. [9]

Q3.

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 4 & -5 & 3 \\ 3 & -4 & 3 \\ 1 & -1 & 2 \end{pmatrix}.$$

Show that $\mathbf{e} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$ and state the corresponding eigenvalue. [2]

Find the other two eigenvalues of $\mathbf{A}$. [4]

The matrix $\mathbf{B}$ is given by

$$\mathbf{B} = \begin{pmatrix} -1 & 4 & 0 \\ -1 & 3 & 1 \\ 1 & -1 & 3 \end{pmatrix}.$$

Show that $\mathbf{e}$ is an eigenvector of $\mathbf{B}$ and deduce an eigenvector of the matrix $\mathbf{AB}$, stating the corresponding eigenvalue. [3]

Matrices 1

Q4.

The matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 10 & -7 & 10 \\ 7 & -5 & 8 \end{pmatrix},$$

has eigenvalues 1 and 3. Find corresponding eigenvectors. [3]

It is given that $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$. Find the corresponding eigenvalue. [2]

Find a diagonal matrix $\mathbf{D}$ and matrices $\mathbf{P}$ and $\mathbf{P}^{-1}$ such that $\mathbf{P}^{-1}\mathbf{AP} = \mathbf{D}$. [5]

Q5.

(i) The vector $\mathbf{e}$ is an eigenvector of the matrix $\mathbf{A}$, with corresponding eigenvalue λ, and is also an eigenvector of the matrix $\mathbf{B}$, with corresponding eigenvalue μ. Show that $\mathbf{e}$ is an eigenvector of the matrix $\mathbf{AB}$ with corresponding eigenvalue $\lambda\mu$. [3]

(ii) Find the eigenvalues and corresponding eigenvectors of the matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 3 \\ 3 & 2 & -3 \\ 1 & 1 & 2 \end{pmatrix}.$$ [6]

(iii) The matrix $\mathbf{B}$, where

$$\mathbf{B} = \begin{pmatrix} 3 & 6 & 1 \\ 1 & -2 & -1 \\ 6 & 6 & -2 \end{pmatrix},$$

has eigenvectors $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. Find the eigenvalues of the matrix $\mathbf{AB}$, and state corresponding eigenvectors. [4]

Q6.

The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{pmatrix} 2 & m & 1 \\ 0 & m & 7 \\ 0 & 0 & 1 \end{pmatrix},$$

where $m \neq 0, 1, 2$.

(i) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{M} = \mathbf{PDP}^{-1}$. [7]

(ii) Find $\mathbf{M}^7\mathbf{P}$. [3]

Matrices 1

Q7.

(a) Find the values of a for which the system of equations

$$\begin{aligned} 3x + y + z &= 0, \\ ax + 6y - z &= 0, \\ ay - 2z &= 0, \end{aligned}$$

does not have a unique solution. [3]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 6 & -1 \\ 0 & 0 & -2 \end{pmatrix}.$$

(b) Use the characteristic equation of $\mathbf{A}$ to find the inverse of $\mathbf{A}^2$. [4]

(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{PDP}^{-1}$. [7]

Q8.

(a) Show that the system of equations

$$\begin{aligned} x - 2y - 4z &= 1, \\ x - 2y + kz &= 1, \\ -x + 2y + 2z &= 1, \end{aligned}$$

where k is a constant, does not have a unique solution. [2]

(b) Given that $k = -4$, show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

(c) Given instead that $k = -2$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

(d) For the case where $k \neq -2$ and $k \neq -4$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]
