Matrices 2

Q1.

The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

(a) State the eigenvalues of $\mathbf{P}$. [1]

(b) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$. [4]

The 3×3 matrix $\mathbf{A}$ has distinct eigenvalues b, -1, 1 with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix},$$

respectively.

(c) Find $\mathbf{A}$ in terms of b. [4]

Q2.

It is given that a is a positive constant.

(a) Show that the system of equations

$$\begin{aligned} ax + (2a+5)y + (a+1)z &= 1, \\ -4y \qquad &= 2, \\ 3y - z &= 3, \end{aligned}$$

has a unique solution and interpret this situation geometrically. [3]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} a & 2a+5 & a+1 \\ 0 & -4 & 0 \\ 0 & 3 & -1 \end{pmatrix}.$$

(b) Show that the eigenvalues of $\mathbf{A}$ are a, -1 and -4. [2]

(c) Find a matrix $\mathbf{P}$ such that

$$\mathbf{A} = \mathbf{P}\begin{pmatrix} a & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{pmatrix}\mathbf{P}^{-1}.$$ [5]

(d) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$. [6]

Matrices 2

Q3.

(a) Given that a is an integer, show that the system of equations

$$\begin{aligned} ax+3y+\ z &= 14, \\ 2x+\ y+3z &= 0, \\ -x+2y-5z &= 17, \end{aligned}$$

has a unique solution and interpret this situation geometrically. [4]

(b) Find the value of a for which $x=1,\ y=4,\ z=-2$ is the solution to the system of equations in part **(a)**. [1]

Q4.

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 5 & -\frac{22}{3} & 8 \\ 0 & -6 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^2 = \mathbf{PDP}^{-1}$. [7]

(b) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^3$. [4]

Q5.

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Use the characteristic equation of $\mathbf{A}$ to show that

$$\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I},$$

where p and q are integers to be determined. [6]

Matrices 2

Q6.

(a) Find the value of a for which the system of equations

$$\begin{aligned} 13x + 18y - 28z &= 0, \\ -4x - ay + 8z &= 0, \\ 2x + 6y - 5z &= 0, \end{aligned}$$

does not have a unique solution. [2]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 13 & 18 & -28 \\ -4 & -1 & 8 \\ 2 & 6 & -5 \end{pmatrix}.$$

(b) Find the eigenvalue of $\mathbf{A}$ corresponding to the eigenvector $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$. [1]

(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} = \mathbf{PDP}^{-1}$. [8]

(d) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$ in terms of $\mathbf{A}$. [2]

Q7.

(a) Show that the system of equations

$$\begin{aligned} x - y + 2z &= 4, \\ x - y - 3z &= a, \\ x - y + 7z &= 13, \end{aligned}$$

where a is a constant, does not have a unique solution. [2]

(b) Given that $a = -5$, show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

(c) Given instead that $a \neq -5$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]
