Momentum 1 MS

Q1.

4	Equate vertical speeds to zero (here $\tan \alpha = 4/3$):	$u \sin \alpha - gt = 0 = ku \cos \alpha - gt$			
	or:	$(u \sin \alpha)^2 - 2gs = (ku \cos \alpha)^2 - 2gs$			
	or equate vertical distances at collision:	$ut \sin \alpha - ½gt^2 = kut \cos \alpha - ½gt^2$	M1 A1		
	Simplify:	$u \sin \alpha = ku \cos \alpha$	A1		
	Evaluate k:	$k = 4/3$	M1 A1	5	
	Find time t to reach ground:	$t = (u \sin \alpha) / g = 4u/5g$	B1		
	Find speed of separation (ignore sign):	$v_P - v_Q = -e(u_P - u_Q)$	M1		
	Substitute for u_P, u_Q:	$v_P - v_Q = -e(u \cos \alpha + ku \sin \alpha)$			
	(ignore sign)	$= -eu(3 + 4k)/5$ $[= -5eu/3]$	A1		
	Find distance apart:	$\lvert v_P - v_Q \rvert t = 4eu^2/3g$	M1 A1	5	**[10]**

Q2.

4	**(i)**	Use conservation of momentum:	$0{\cdot}1 v_A + m v_B = 0{\cdot}1 \times 5 - m \times 2$	M1		
		Find m:	$m = (0{\cdot}5 - 0{\cdot}1 v_A) / (2 + v_B)$	A1		
		Use $v_A > 0$ to find lower bound on m:	$v_B > 0$, $m < 0{\cdot}5/2 = 0{\cdot}25$ **A.G.**	M1 A1	4	
	(ii)	Use Newton's law of restitution:	$v_B - v_A = ½(2 + 5) = 7/2$	M1 A1		
		Put $m = 0{\cdot}2$ and find one of v_A, v_B:	$2 v_B + v_A = 1$, $v_A = -2$ *or* $v_B = 1{\cdot}5$	M1 A1		
		Find magnitude of impulse [N s]:	$0{\cdot}1 (5 - v_A)$ *or* $0{\cdot}2 (1{\cdot}5 + 2) = 0{\cdot}7$	M1 A1	6	**[10]**

Q3.

3	Use conservation of momentum:	$mv_A + 3mv_B = mu$	M1		
	Use Newton's law of restitution:	$v_A - v_B = -eu$	M1		
	Solve for v_A:	$v_A = ¼(1 - 3e)u$	M1 A1		
	Use $e > ⅓$ to find direction of A:	$1 < 3e$ so $v_A < 0$ **A.G.**	B1	(5)	
	Find speed of B before striking barrier:	$v_B = ¼(1 + e)u$	A1		
	Find rel. speed or ratio of speeds after collision:	$e v_B + v_A = ¼(1 - e)^2 u$			
	or	$-v_A / e v_B = 1 - (1 - e)^2 / e(1 + e)$	M1		
	Derive condition for subsequent collision:	$e v_B > -v_A$ unless $e = 1$ **A.G.**	A1	(3)	**[8]**

Momentum 1 MS

Q4.

4	Use conservation of momentum for 1st collision: $3mu_A + mu_B = 3mu$	M1		
	Use Newton's law of restitution (A1 if both correct): $u_A - u_B = -0{\cdot}6\,u$	M1 A1		
	Solve for u_A and u_B: $u_A = 0{\cdot}6\,u$ and $u_B = 1{\cdot}2u$	M1 A1	5	
	Find speed u'_B of B after striking wall: $u'_B = eu_B$ [$= 1{\cdot}2eu$]	M1		
	Use conservation of momentum for 2nd collision: $mv_B = 3mu_A - mu'_B$	B1		
	Use Newton's law of restitution: $v_B = 0{\cdot}6\,(u_A + u'_B)$	B1		
	Combine to find e: $1{\cdot}6e \times 1{\cdot}2u = 2{\cdot}4 \times 0{\cdot}6u,\ e = ¾$	M1 A1	5	**[10]**

Q5.

4	Use conservation of momentum: $3mv_Q = mu + 3kmu$	M1 A1		
	Use Newton's law of restitution: $v_Q = e(u - ku)$	M1 A1		
	Eliminate v_Q to find e: $e = (3k + 1)/3(1 - k)$ **A.G.**	M1 A1	6	
	Relate K.E. after and before collision: $\frac{1}{2}\,3mv_Q^2 = \frac{2}{3}\,\frac{1}{2}m\,(u^2 + 3k^2u^2)$	M1 A1		
	Replace v_Q by $\frac{1}{3}(1 + 3k)u$ and rearrange: $(1 + 3k)^2 = 2(1 + 3k^2)$			
	$3k^2 + 6k - 1 = 0$	M1 A1		
	Find root k with $0 < k < 1$: **A.G.** $k = (-6 + \sqrt{48})/6 = \frac{1}{3}(2\sqrt{3} - 3)$	A1	5	
	(Simply substituting given k earns M1 A0 A1)			**[11]**

Q6.

2	*EITHER:*	Find/ state comps. of speed after 1st colln.: $u\cos\alpha$ // to A and $eu\sin\alpha \perp$ to A	M1 A1		
		Find/ state comps. of speed after 2nd colln.: $eu\sin\alpha$ // to B and $eu\cos\alpha \perp$ to	A1		
		Find final angle with barrier B: **AG** $\tan^{-1}(eu\cos\alpha / eu\sin\alpha) = \frac{1}{2}\pi - \alpha$	A1		
	OR:	Relate angle β_1 after 1st colln. to α: $v_1\sin\beta_1 = eu\sin\alpha$			
		$v_1\cos\beta_1 = u\cos\alpha$			
		$\tan\beta_1 = e\tan\alpha$	(M1 A1)		
		Relate angle β_2 after 2nd colln. to β_1: $v_2\sin\beta_2 = ev_1\cos\beta_1$			
		$v_2\cos\beta_2 = v_1\sin\beta_1$			
		$\tan\beta_2 = e / \tan\beta_1$	(A1)		
		Find final angle β_2 with barrier B: **AG** $\tan^{-1}(e / e\tan\alpha) = \frac{1}{2}\pi - \alpha$	(A1)	4	
	EITHER:	Find total loss or gain in KE, e.g.: $\frac{1}{2}m\{u^2 - (eu\sin\alpha)^2 - (eu\cos\alpha)^2\}$	M1 A1		
		Find total loss in KE (A0 if wrong sign): $= \frac{1}{2}m(1 - e^2)u^2$	A1		
	OR:	Find or state final speed u_{final}: $u_{final} = eu$	(B1)		
		Find total loss in KE (A0 if wrong sign): $\frac{1}{2}m(u^2 - u_{final}^2) = \frac{1}{2}m(1 - e^2)u^2$	(M1 A1)	3	**7**

Q7.

6(a)	$mu = mw + 2mv$	**B1**	Momentum equation (with m)
	$v - w = eu$	**B1**	Restitution with consistent signs
	$v = \frac{u}{3}(e+1)$ $w = \frac{u}{3}(1-2e)$	**B1**	Both correct.
		3	

Momentum 1 MS

6(b)	Perpendicular to plane: $y = ev\sin\theta$ Parallel to plane: $x = v\cos\theta$	**B1**	Both
	Speed of $B = \sqrt{x^2+y^2} = \sqrt{v^2\left(\left(\frac{4}{5}\right)^2 + \left(\frac{2}{3}\cdot\frac{3}{5}\right)^2\right)}$ $\left(= \frac{2}{\sqrt{5}}v\right)$	**M1**	Speed of B
	KE of $B = \frac{1}{2}.2m\frac{4}{5}.\frac{u^2}{9}(e+1)^2$	**M1**	KE of B in terms of u. $\frac{1}{2}$ and $2m$ needed
	KE of $A = \frac{1}{2}.m.\frac{u^2}{9}(1-2e)^2$ So $\frac{1}{2}.m.\frac{u^2}{9}(1-2e)^2 = \frac{5}{32}.\frac{1}{2}.2m\frac{4}{5}.\frac{u^2}{9}(e+1)^2$	**M1 A1**	Relate the two KEs
	$4(1-2e)^2 = (e+1)^2$ or $15e^2 - 18e + 3 = 0$	**M1**	Rearrange and simplify to quadratic
	$1+e = \pm 2(1-2e)$ $e = \frac{1}{5}$, 1	**A1**	Both values
		7	

Q8.

6(a)	Along line of centres, speeds v_1 and v_2 $mv_1 + mv_2 = mu\cos\alpha - mu\cos\beta$	**M1**	Momentum (condone missing masses).
	$v_2 - v_1 = eu(\cos\beta + \cos\alpha)$	**M1**	Restitution.
	Both correct, masses seen.	**A1**	
	$v_1 = 0$ so A has no speed along line of centres: moves perpendicular to line of centres	**A1**	AG.
		4	
6(b)	$(v_2 = \frac{1}{2}u\cos\alpha = u\cos\beta)$ KE of B after collision is $\frac{1}{2}m\left(v_2^2 + (u\sin\beta)^2\right)$ KE of A after collision $= \frac{1}{2}m(u\sin\alpha)^2$	**M1**	Both components.
	Add both KEs and equate to $\frac{3}{4}mu^2$	**M1**	
	Simplify to equation in $\sin\alpha$	**M1**	
	$\sin\alpha = \frac{1}{\sqrt{2}}$, $\alpha = 45°$	**A1**	
		4	