Momentum 1

Q1.

Two identical smooth balls, P and Q, are projected simultaneously towards each other from two points on horizontal ground. P is projected with speed u at an angle $\tan^{-1}\left(\frac{4}{3}\right)$ to the horizontal and Q is projected with speed ku at an angle $\tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal (see diagram). The balls collide when they are moving horizontally. It may be assumed that there is no air resistance. Find the value of k. [5]

The coefficient of restitution between the balls is e. Find, in terms of e, u and g, the distance between the points where the balls first hit the ground. [5]

Q2.

Two smooth spheres A and B, of equal radii, have masses 0.1 kg and m kg respectively. They are moving towards each other in a straight line on a smooth horizontal table and collide directly. Immediately before collision the speed of A is $5\,\text{m s}^{-1}$ and the speed of B is $2\,\text{m s}^{-1}$.

(i) Assume that in the collision A does not change direction. The speeds of A and B after the collision are $v_A\,\text{m s}^{-1}$ and $v_B\,\text{m s}^{-1}$ respectively. Express m in terms of v_A and v_B, and hence show that $m < 0.25$. [4]

(ii) Assume instead that $m = 0.2$ and that the coefficient of restitution between the spheres is $\frac{1}{2}$. Find the magnitude of the impulse acting on A in the collision. [6]

Q3.

Two small smooth spheres A and B of equal radius have masses m and $3m$ respectively. They lie at rest on a smooth horizontal plane with their line of centres perpendicular to a smooth fixed vertical barrier with B between A and the barrier. The coefficient of restitution between A and B, and between B and the barrier, is e, where $e > \frac{1}{3}$. Sphere A is projected directly towards B with speed u. Show that after colliding with B the direction of motion of A is reversed. [5]

After the impact, B hits the barrier and rebounds. Show that B will subsequently collide with A again unless $e = 1$. [3]

Momentum 1

Q4.

Two uniform spheres A and B, of equal radius, are at rest on a smooth horizontal table. Sphere A has mass $3m$ and sphere B has mass m. Sphere A is projected directly towards B, with speed u. The coefficient of restitution between the spheres is 0.6. Find the speeds of A and B after they collide. [5]

Sphere B now strikes a wall that is perpendicular to its path, rebounds and collides with A again. The coefficient of restitution between B and the wall is e. Given that the second collision between A and B brings A to rest, find e. [5]

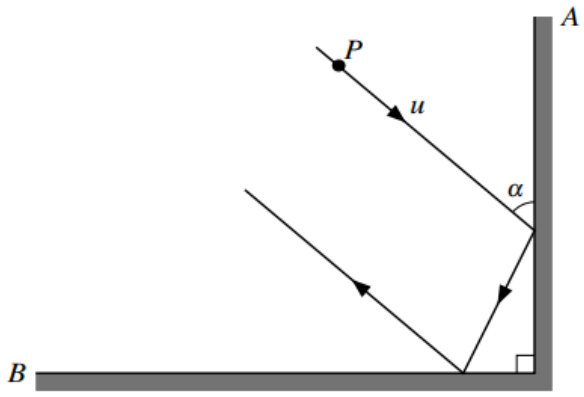
Q5.

Two smooth spheres P and Q, of equal radius, have masses m and $3m$ respectively. They are moving in the same direction in the same straight line on a smooth horizontal table. Sphere P has speed u and collides directly with sphere Q which has speed ku, where $0 < k < 1$. Sphere P is brought to rest by the collision. Show that the coefficient of restitution between P and Q is $\frac{3k+1}{3(1-k)}$. [6]

One third of the total kinetic energy of the spheres is lost in the collision. Show that

$$k = \tfrac{1}{3}(2\sqrt{3} - 3).$$ [5]

Q6.



A uniform sphere P of mass m is at rest on a smooth horizontal table. The sphere is projected along the table with speed u and strikes a smooth vertical barrier A at an acute angle α. It then strikes another smooth vertical barrier B which is at right angles to A (see diagram). The coefficient of restitution between P and each of the barriers is e. Show that the final direction of motion of P makes an angle $\frac{1}{2}\pi - \alpha$ with the barrier B and find the total loss in kinetic energy as a result of the two impacts. [7]

Momentum 1

Q7.

Two smooth spheres A and B have equal radii and masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal floor. Sphere A is moving on the floor with velocity u and collides directly with B. The coefficient of restitution between the spheres is e.

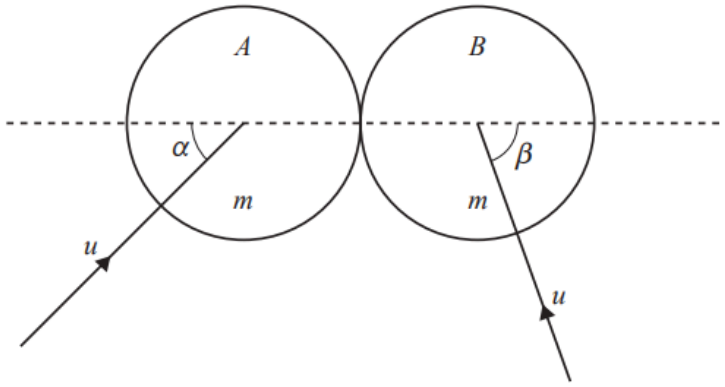
(a) Find, in terms of u and e, the velocities of A and B after the collision. [3]

Subsequently, B collides with a fixed vertical wall which makes an angle θ with the direction of motion of B, where $\tan\theta = \frac{3}{4}$.

The coefficient of restitution between B and the wall is $\frac{2}{3}$. Immediately after B collides with the wall, the kinetic energy of A is $\frac{5}{32}$ of the kinetic energy of B.

(b) Find the possible values of e. [7]

Q8.



Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision, A's direction of motion makes an angle α with the line of centres, and B's direction of motion makes an angle β with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$ and $2\cos\beta = \cos\alpha$.

(a) Show that the direction of motion of A after the collision is perpendicular to the line of centres. [4]

The total kinetic energy of the spheres after the collision is $\frac{3}{4}mu^2$.

(b) Find the value of α. [4]
