

Trigonometry 2

Q1.

(i) Prove the identity $\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} \equiv \frac{\tan \theta - 1}{\tan \theta + 1}$. [1]

(ii) Hence solve the equation $\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} = \frac{\tan \theta}{6}$, for $0^\circ \leq \theta \leq 180^\circ$. [4]

Q2.

(i) Express the equation $3 \sin \theta = \cos \theta$ in the form $\tan \theta = k$ and solve the equation for $0^\circ < \theta < 180^\circ$. [2]

(ii) Solve the equation $3 \sin^2 2x = \cos^2 2x$ for $0^\circ < x < 180^\circ$. [4]

Q3.

(i) Show that the equation $\frac{4 \cos \theta}{\tan \theta} + 15 = 0$ can be expressed as $4 \sin^2 \theta - 15 \sin \theta - 4 = 0$. [3]

(ii) Hence solve the equation $\frac{4 \cos \theta}{\tan \theta} + 15 = 0$ for $0^\circ \leq \theta \leq 360^\circ$. [3]

Q4.

(i) Prove the identity $\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 \equiv \frac{1 - \cos x}{1 + \cos x}$. [4]

(ii) Hence solve the equation $\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 = \frac{2}{5}$ for $0 \leq x \leq 2\pi$. [3]

Q5.

(a) Solve the equation $\sin^{-1}(3x) = -\frac{1}{3}\pi$, giving the solution in an exact form. [2]

(b) Solve, by factorising, the equation $2 \cos \theta \sin \theta - 2 \cos \theta - \sin \theta + 1 = 0$ for $0 \leq \theta \leq \pi$. [4]

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Q6.

The function f is defined by $f : x \mapsto 4 \sin x - 1$ for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$.

- (i) State the range of f . [2]
 - (ii) Find the coordinates of the points at which the curve $y = f(x)$ intersects the coordinate axes. [3]
 - (iii) Sketch the graph of $y = f(x)$. [2]
 - (iv) Obtain an expression for $f^{-1}(x)$, stating both the domain and range of f^{-1} . [4]
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Q7.

(i) Prove the identity $\frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} \equiv \frac{4}{\sin \theta \tan \theta}$. [4]

(ii) Hence solve, for $0^\circ < \theta < 360^\circ$, the equation

$$\sin \theta \left(\frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} \right) = 3. \quad [3]$$

Q8.

- (i) Show that $3 \sin x \tan x - \cos x + 1 = 0$ can be written as a quadratic equation in $\cos x$ and hence solve the equation $3 \sin x \tan x - \cos x + 1 = 0$ for $0 \leq x \leq \pi$. [5]
 - (ii) Find the solutions to the equation $3 \sin 2x \tan 2x - \cos 2x + 1 = 0$ for $0 \leq x \leq \pi$. [3]
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Q9.

- (i) Show that $\cos^4 x \equiv 1 - 2 \sin^2 x + \sin^4 x$. [1]
 - (ii) Hence, or otherwise, solve the equation $8 \sin^4 x + \cos^4 x = 2 \cos^2 x$ for $0^\circ \leq x \leq 360^\circ$. [5]
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Q10.

- (i) Express the equation $\sin 2x + 3 \cos 2x = 3(\sin 2x - \cos 2x)$ in the form $\tan 2x = k$, where k is a constant. [2]
 - (ii) Hence solve the equation for $-90^\circ \leq x \leq 90^\circ$. [3]
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