Algebra 1

Q1.

Solve the inequality $|x + 3a| > 2|x - 2a|$, where a is a positive constant. [4]

Q2.

(i) Express $\dfrac{2}{(x+1)(x+3)}$ in partial fractions. [2]

(ii) Using your answer to part **(i)**, show that

$$\left(\frac{2}{(x+1)(x+3)}\right)^2 \equiv \frac{1}{(x+1)^2} - \frac{1}{x+1} + \frac{1}{x+3} + \frac{1}{(x+3)^2}.$$ [2]

Q3

Solve the equation

$$\frac{2^x + 1}{2^x - 1} = 5,$$

giving your answer correct to 3 significant figures. [4]

Q4.

The polynomial $2x^3 + 5x^2 + ax + b$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 1)$ is a factor of $\mathrm{p}(x)$ and that when $\mathrm{p}(x)$ is divided by $(x + 2)$ the remainder is 9.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $\mathrm{p}(x)$ completely. [3]

Q5

(i) Find the values of the constants A, B, C and D such that

$$\frac{2x^3 - 1}{x^2(2x - 1)} \equiv A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{2x - 1}.$$ [5]

.Q6.

Solve the inequality $|x - 3| > 2|x + 1|$. [4]

Q7.

(i) Express $\dfrac{4+5x-x^2}{(1-2x)(2+x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{4+5x-x^2}{(1-2x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

Q8.

Solve the inequality $2|x-3| > |3x+1|$. [4]

Q9.

Let $f(x) = \dfrac{3x}{(1+x)(1+2x^2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. [5]

Q10.

Expand $(1+2x)^{-3}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [3]

Q11.

The polynomial $p(z)$ is defined by

$$p(z) = z^3 + mz^2 + 24z + 32,$$

where m is a constant. It is given that $(z+2)$ is a factor of $p(z)$.

(i) Find the value of m. [2]

(ii) Hence, showing all your working, find

(a) the three roots of the equation $p(z) = 0$, [5]

(b) the six roots of the equation $p(z^2) = 0$. [6]