Algebra 2 - Marking Scheme

Q1.

<u>Either</u>: Obtain $1+\frac{1}{3}kx$, where $k=\pm 6$ or ± 1 — M1

Obtain $1-2x$ — A1

Obtain $-4x^2$ — A1

Obtain $-\frac{40}{3}x^3$ or equivalent — A1

<u>Or</u>: Differentiate expression to obtain form $k(1-6x)^{-\frac{2}{3}}$ and evaluate f(0) and f′(0) — M1

Obtain $f'(x) = -2(1-6x)^{-\frac{2}{3}}$ and hence the correct first two terms $1-2x$ — A1

Obtain $f''(x) = -8(1-6x)^{-\frac{5}{3}}$ and hence $-4x^2$ — A1

Obtain $f'''(x) = -80(1-6x)^{-\frac{8}{3}}$ and hence $-\frac{40}{3}x^3$ or equivalent — A1 [4]

Q2.

(i) Verify that $-96 + 100 + 8 - 12 = 0$ — B1

Attempt to find quadratic factor by division by $(x + 2)$, reaching a partial quotient $12x^2 + kx$, inspection or use of an identity — M1

Obtain $12x^2 + x - 6$ — A1

State $(x + 2)(4x + 3)(3x - 2)$ — A1 [4]

[The M1 can be earned if inspection has unknown factor $Ax^2 + Bx - 6$ and an equation in A and/or B or equation $12x^2 + Bx + C$ and an equation in B and/or C.]

(ii) State $3^y = \frac{2}{3}$ and no other value — B1

Use correct method for finding y from equation of form $3^y = k$, where $k > 0$ — M1

Obtain -0.369 and no other value — A1 [3]

Q3.

EITHER: State or imply non-modular inequality $x^2 < (5+2x)^2$, or corresponding equation, or pair of linear equations $x = \pm(5+2x)$ — M1

Obtain critical values -5 and $-\frac{5}{3}$ only — A1

Obtain final answer $x < -5$, $x > -\frac{5}{3}$ — A1

OR: State one critical value e.g. -5, by solving a linear equation or inequality, or from a graphical method, or by inspection — B1

State the other critical value, e.g. $-\frac{5}{3}$, and no other — B1

Obtain final answer $x < -5$, $x > -\frac{5}{3}$ — B1 [3]

[Do not condone $\le$ or $\ge$.]

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Q4.

(i) State or imply partial fractions are of the form $\frac{A}{1+x} + \frac{Bx+C}{2+x^2}$ — B1

Use a relevant method to determine a constant — M1

Obtain one of the values $A = -2$, $B = 1$, $C = 4$ — A1

Obtain a second value — A1

Obtain the third value — A1 [5]

(ii) Use correct method to obtain the first two terms of the expansion of $(1+x)^{-1}$, $\left(1+\frac{1}{2}x^2\right)^{-1}$ or $(2+x^2)^{-1}$ in ascending powers of x — M1

Obtain correct unsimplified expansion up to the term in x^3 of each partial fraction — A1√ + A1√

Multiply out fully by $Bx + C$, where $BC \neq 0$ — M1

Obtain final answer $\frac{5}{2}x - 3x^2 + \frac{7}{4}x^3$, or equivalent — A1 [5]

[Symbolic binomial coefficients, e.g. $\binom{-1}{1}$, are not sufficient for the first M1. The f.t. is on *A, B, C*.]

[If *B* or *C* omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**, max 4/10.]

[In the case of an attempt to expand $(5x - x^2)(1 + x)^{-1}(2 + x^2)^{-1}$, give M1A1A1 for the expansions, M1 for the multiplying out fully, and A1 for the final answer.]

[Allow use of Maclaurin, giving M1A1√A1√ for differentiating and obtaining $f(0) = 0$ and $f'(0) = \frac{5}{2}$, A1√ for $f''(0) = -6$, and A1 for $f'''(0) = \frac{21}{2}$ and the final answer (the f.t. is on *A*, *B*, *C* if used).]

[For the identity $5x - x^2 \equiv (2 + 2x + x^2 + x^3)(a + bx + cx^2 + dx^3)$ give M1A1; then M1A1 for using a relevant method to obtain two of $a = 0$, $b = \frac{5}{2}$, $c = -3$ and $d = \frac{7}{4}$; then A1 for the final answer in series form.]

Q5.

(i) Substitute $x = \frac{1}{2}$ and equate to zero, or divide, and obtain a correct equation, e.g. $\frac{1}{8}a + \frac{1}{4}b + \frac{5}{2} - 2 = 0$ — B1

Substitute $x = 2$ and equate result to 12, or divide and equate constant remainder to 12 — M1

Obtain a correct equation, e.g. $8a + 4b + 10 - 2 = 12$ — A1

Solve for a or for b — M1

Obtain $a = 2$ and $b = -3$ — A1 [5]

(ii) Attempt division by $2x - 1$ reaching a partial quotient $\frac{1}{2}ax^2 + kx$ — M1

Obtain quadratic factor $x^2 - x + 2$ — A1 [2]

[The M1 is earned if inspection has an unknown factor $Ax^2 + Bx + 2$ and an equation in *A* and/or *B*, or an unknown factor of $\frac{1}{2}ax^2 + Bx + C$ and an equation in *B* and/or *C*.]

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Q6.

(i)	*EITHER*:	Attempt division by $x^2 - x + 1$ reaching a partial quotient of $x^2 + kx$	M1	
		Obtain quotient $x^2 + 4x + 3$	A1	
		Equate remainder of form lx to zero and solve for a, or equivalent	M1	
		Obtain answer $a = 1$	A1	
	OR:	Substitute a complex zero of $x^2 - x + 1$ in p(x) and equate to zero	M1	
		Obtain a correct equation in a in any unsimplified form	A1	
		Expand terms, use $i^2 = -1$ and solve for a	M1	
		Obtain answer $a = 1$	A1	[4]

[SR: The first M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B. The second M1 is only earned if use of the equation $a = B - C$ is seen or implied.]

(ii)	State answer, e.g. $x = -3$	B1	
	State answer, e.g. $x = -1$ and no others	B1	[2]

Q7.

(i)	Use any relevant method to determine a constant	M1	
	Obtain one of the values $A = 3$, $B = 4$, $C = 0$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	[4]

Q8.

1	**Either**		
	Obtain correct unsimplified version of x or x^2 term in expansion of $(2 + x)^{-2}$ or $(1 + \frac{1}{2}x)^{-2}$	M1	
	Correct first term 4 from correct work	B1	
	Obtain $-4x$	A1	
	Obtain $+ 3x^2$	A1	
	Or		
	Differentiate and evaluate f(0) and f′ (0) where $f'(x) = k(2+x)^{-3}$	M1	
	State correct first term 4	B1	
	Obtain $-4x$	A1	
	Obtain $+ 3x^2$	A1	[4]

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Q9.

7	**(i)**	Substitute $x = \frac{1}{2}$ and equate to zero		
		or divide by $(2x-1)$, reach $\frac{a}{2}x^2 + kx + ...$and equate remainder to zero		
		or by inspection reach $\frac{a}{2}x^2 + bx + c$ and an equation in b/c		
		or by inspection reach $Ax^2 + Bx + a$ and an equation in A/B	M1	
		Obtain $a = 2$	A1	
		Attempt to find quadratic factor by division or inspection or equivalent	M1	
		Obtain $(2x-1)(x^2+2)$	A1cwo	[4]
	(ii)	State or imply form $\frac{A}{2x-1} + \frac{Bx+C}{x^2+2}$, following factors from part **(i)**	B1√	
		Use relevant method to find a constant	M1	
		Obtain $A = -4$, following factors from part **(i)**	A1√	
		Obtain $B = 2$	A1	
		Obtain $C = 5$	A1	

Q10.

(i)	Either	Obtain correct (unsimplified) version of x or x^2 term from $(1-4x)^{\frac{1}{2}}$	M1	
		Obtain $1 + 2x$	A1	
		Obtain $+ 6x^2$	A1	
	Or	Differentiate and evaluate f(0) and f′(0) where $f'(x) = k(1-4x)^{-\frac{3}{2}}$	M1	
		Obtain $1 + 2x$	A1	
		Obtain $+ 6x^2$	A1	[3]
(ii)	Combine both x^2 terms from product of $1 + 2x$ and answer from part **(i)**		M1	
	Obtain 5		A1	[2]