Continuous Random Variables 2

Q1.

6 (i)	$\int_4^{25} kx^{-\frac{1}{2}}\,dx = 1$	M1	Attempt integrate & = 1. Ignore limits
	$\left[\frac{kx^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^{25} = 1$		
	$2k(5-2) = 1$ ($k = \frac{1}{6}$ **AG**)	A1 [2]	or equiv correct subst of correct limits
(ii)	$\frac{1}{6}\int_4^{25} x^{\frac{1}{2}}\,dx$	M1	Attempt integ $xf(x)$. Ignore limits
	$= \frac{1}{6}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_4^{25}$ $(= \frac{1}{9}(125-8)$	A1	Correct integrand and limits
	$= 13$	A1 [3]	Or 117/9
(iii)	$\frac{1}{6}\int_{20}^{25} x^{-\frac{1}{2}}\,dx$	M1	Attempt integ f(x) from 20 to 25 Or $1 - \int_4^{20}$
	$(= \frac{1}{6}\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_{20}^{25} = \frac{1}{3}(5-\sqrt{20}))$		
	$= 0.176$ (3 sfs)	A1 [2]	Accept surd form
(iv)	Wkly demand may be > 25 (or < 4)	B1 [1]	or other sensible

Q2.

1 $(\frac{m}{2})^2$	M1	$y = \frac{1}{2}x$ (attempt at linear equ with c = 0)
$(\frac{m}{2})^2 = \frac{1}{2}$	M1	$\int_0^m (\frac{1}{2}x)\,dx = \frac{1}{2}$
$m = \sqrt{2}$ or 1.41 (3 sfs)	A1 [3]	(Note: $\pm\sqrt{2}$ as final answer scores A0)

Q3.

Continuous Random Variables 2

6 **(i)**	f(x) $\geqslant$ 0 for all x defined	B1		
	$\int_0^a \frac{2}{a^2} x\mathrm{d}x$	M1		Attempt $\int \mathrm{f}(x)\mathrm{d}x$ with limits 0, a . Must be a.
	$\left(=\left[\frac{2x^2}{2a^2}\right]_0^a\right)$	A1		Or equivalent methods (e.g. by areas)
	$= 1$		[3]	
(ii)	$\int_0^a \frac{2}{a^2} x^2\mathrm{d}x \ (= 8)$	M1		Attempt $\int x\mathrm{f}(x)\mathrm{d}x$, ignore limits
	$\frac{2}{a^2}\left[\frac{x^3}{3}\right]_0^a \ (= 8)$	A1		Correct integrand and limits
	$\left(\frac{2a}{3} = 8\right)$	A1		
	$a = 12$		[3]	
(iii)	$1 - \int_0^6 \frac{2}{144} x\mathrm{d}x$ or $\int_6^{12} \frac{2}{144} x\mathrm{d}x$	M1		Correct expr'n incl limits; ft their 'a'
	$=1 - 1 - 1 - \frac{1}{72}\left[\frac{x^2}{2}\right]_0^6$ or $\frac{1}{72}\left[\frac{x^2}{2}\right]_6^{12}$	A1ft		Correct integrand and limits; ft their 'a'
	$= \frac{3}{4}$	A1ft	[3]	ft their 'a', dep 0 < ans < 1

Q4.

2 **(i)**	$\frac{2}{3}\int_1^2 x^2\mathrm{d}x$	M1	Attempt integ. $x\mathrm{f}(x)$; ignore limits
	$= \frac{2}{3}\left[\frac{x^3}{3}\right]_1^2$	A1	Correct integration and limits
	$= \frac{14}{9}$ or 1.56 o.e.	A1 [3]	
(ii)	$\frac{2}{3}\int_1^{\frac{14}{9}} x\,dx$	M1	Attempt integ. f(x); with limits
	$\left(= \frac{2}{3}\left[\frac{x^3}{3}\right]_1^2\right)$		
	$= \frac{115}{243}$ or 0.473 (3 s.f.)	A1 [2]	
(iii)	$\frac{115}{243} < \frac{1}{2}$ o.e.	M1	Comparison of prob. or values
	Hence mean < median	A1ft[2]	ft (i) or (ii)

Continuous Random Variables 2

Q5.

5 (i) $\int_0^2 k(x-2)^2\,dx = 1$	M1	Attempt to integrate f(x) with correct limits and = 1
$\left(\left[\frac{k(x-2)^3}{3}\right]_0^2 = 1\right)$		
$k\,[0-(-\frac{8}{3})] = 1$	A1	Must see this line or better, e.g. $k \times \frac{8}{3} = 1$
$k = \frac{3}{8}$ **AG**	[2]	
(ii) $\frac{3}{8}\int_d^2 (x-2)^2\,dx = 0.2$	M1	$\int$f(x)dx with limits d and 2 or 0 and d, and = 0.2 or =0.8 Condone missing 'k'
$\left(\frac{3}{8}\left[\frac{(x-2)^3}{3}\right]_d^2 = 0.2\right)$		
$\frac{3}{8}\left[0-\frac{(d-2)^3}{3}\right] = 0.2$ oe	M1	Reasonable attempt to integrate from a correct expression, with limits substituted to give expression in d^3. Condone missing 'k'
$((d-2)^3 = -1.6)$		
$d = 0.83(0)$ (3 s.f.)	A1 [3]	
(iii) $\frac{3}{8}\int_0^2 x(x-2)^2\,dx$	M1	Attempt integ xf(x); ignore limits, condone missing k $\left(\frac{3}{8}\left[x\times\frac{(x-2)^3}{3} - \int\frac{(x-2)^3}{3}dx\right]_0^2\right)$
$\left(= \frac{3}{8}\int_0^2 x^3 - 4x^2 + 4x\,dx\right)$		
$= \frac{3}{8}\left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2\right]_0^2$	A1	$= \frac{3}{8}\left[x\times\frac{(x-2)^3}{3} - \frac{(x-2)^4}{12}\right]_0^2$ Correct integration & limits, condone missing k
$= \frac{1}{2}$	A1 [3]	

Continuous Random Variables 2

Q6.

3 (i)	$\int_0^{10} \frac{1}{2500}\left(100t^3 - t^5\right)dt$	**M1**		Attempt integ t^2f(t)
	$\left(= \frac{1}{2500}\left[25t^4 - \frac{t^6}{6}\right]_0^{10} = \frac{100}{3}\right)$			
	"$\frac{100}{3}$" $- \left(\frac{16}{3}\right)^2$	**M1**		For E (T^2) – (E (T))^2
	$= \frac{44}{9}$ or 4.89 (3 sf)	**A1**	**3**	
(ii)	$\int_n^{10} \frac{1}{2500}\left(100t - t^3\right)dt$	**M1**		Attempt integ f(t), ignore limits
	$\frac{1}{2500}\left[50t^2 - \frac{t_4}{4}\right] = 0.1$	**M1**		Attempt integ f(t), limits n to 10 or 0 to n Equated to 0.1 or 0.9. Not need to be matched
	$\frac{1}{2500}\left[2500 - \left(50n^2 - \frac{n^4}{4}\right)\right] = 0.1$	**M1**		0.1/0.9 matched to correct limits and used
	$(n^4 - 200n^2 + 9000 = 0)$	**M1**		Correct method of solution of a QE in n^2
	$(n^2 = 68.3772, n = 8.27)$ $n = 8$	**A1**	**5**	Must be single ans only
Total			**[8]**	

Q7.

1	$\frac{1}{2}a^2 = 1$	M1		or $\int_0^a x\,dx = 1$
	$a = \sqrt{2}$	A1		Allow 1.41 or better
	$\int_0^{\sqrt{2}} x^2 dx$	M1		ignore limits
	$= \left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$	A1f		correct integral and limits, but ft their a
	$= \frac{\left(\sqrt{2}\right)^3}{3}$ = or $\frac{2^{1.5}}{3}$ or $\frac{2.83}{3}$ or 0.9428 (= 0.943 **AG**)	A1	[5]	must see this numerical expression, or equiv SR Equating $\int x$ f(x) to 0.943 scores M1 Solving to find a = 1.41 scores A1

Continuous Random Variables 2

Q8.

7 (i)	$\frac{3}{4}\int_0^c (cx - x^2)dx = 1$	M1		Attempt integ f(x) and = 1. Ignore limits
	$\frac{3}{4}\left[\frac{cx^2}{2} - \frac{x^3}{3}\right]_0^c = 1$	A1		Correct integration and limits (condone c = 2
	$\frac{3}{4}\left(\frac{c^3}{2} - \frac{c^3}{3}\right) = 1$ or $\frac{3}{4} \times \frac{c^3}{6} = 1$ or $\frac{c^3}{8} = 1$	A1	[3]	No errors seen
	($c = 2$ **AG**)			
(ii)	Inverted parabola	B1		
	Through (0, 0) and (2, 0) and zero elsewhere	B1		Must not extend beyond [0,2]
	Median = 1	B1	[3]	
(iii)	$\frac{3}{4}\int_0^{1.5} (2x - x^2)dx$	M1		Attempt integ f(x) ignore limits
	$= \frac{3}{4}\left[x^2 - \frac{x^3}{3}\right]_0^{1.5}$	A1		Correct integration ignore limits
	$\frac{3}{4}\left(1.5^2 - \frac{1.5^3}{3}\right)$	B1		Use of correct limits [0,1.5] or 1–[1.5,2]
	$= \frac{27}{32}$ or 0.844 (3 sf)	A1	[4]	