Differentiation 2 - Marking Scheme

Q1.

(i) Obtain $2y\frac{dy}{dx}$ as derivative of y^2 — B1

Obtain $-4y-4x\frac{dy}{dx}$ as derivative of $-4xy$ — B1

Substitute $x = 2$ and $y = -3$ and find value of $\frac{dy}{dx}$

(dependent on at least one B1 being earned and $\frac{d(45)}{dx}=0$) — M1

Obtain $\frac{12}{7}$ or equivalent — A1 [4]

(ii) Substitute $\frac{dy}{dx}=1$ in an expression involving $\frac{dy}{dx}$, x and y and obtain $ay = bx$ — M1

Obtain $y = x$ or equivalent — A1

Uses $y = x$ in original equation and demonstrate contradiction — A1 [3]

Q2.

(i) State derivative in any correct form, e.g. $3\cos x-12\cos^2 x\sin x$ — B1 + B1

Equate derivative to zero and solve for sin $2x$, or sin x or cos x — M1

Obtain answer $x=\frac{1}{12}\pi$ — A1

Obtain answer $x=\frac{5}{12}\pi$ — A1

Obtain answer $x=\frac{1}{2}\pi$ and no others in the given interval — A1√ [6]

(ii) Carry out a method for determining the nature of the relevant stationary point — M1

Obtain a maximum at $\frac{1}{12}\pi$ correctly — A1 [2]

[Treat answers in degrees as a misread and deduct A1 from the marks for the angles.]

Q3.

Obtain $\frac{dx}{d\theta}=2\cos 2\theta-1$ or $\frac{dy}{d\theta}=-2\sin 2\theta+2\cos\theta$, or equivalent — B1

Use $\frac{dy}{dx}=\frac{dy}{d\theta}\div\frac{dx}{d\theta}$ — M1

Obtain $\frac{dy}{dx}=\frac{-2\sin 2\theta+2\cos\theta}{2\cos 2\theta-1}$, or equivalent — A1

At any stage use correct double angle formulae throughout — M1

Obtain the given answer following full and correct working — A1 [5]

Differentiation 2 - Marking Scheme

Q4.

(i)	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form, e.g. $\dfrac{2e^{2x}}{x^3} - \dfrac{3e^{2x}}{x^4}$	A1	
	Equate derivative to zero and solve a 2-term equation for non-zero x	M1	
	Obtain $x = \dfrac{3}{2}$ correctly	A1	[4]
(ii)	Carry out a method for determining the nature of a stationary point, e.g. test derivative either side	M1	
	Show point is a minimum with no errors seen	A1	[2]

Q5.

(i)	*EITHER*:	State or imply $\frac{1}{x} + \frac{1}{y}\frac{dy}{dx}$ as derivative of $\ln xy$, or equivalent	B1	
		State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3, or equivalent	B1	
		Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	
		Obtain the given answer	A1	
	OR	Obtain $xy = \exp(1 + y^3)$ and state or imply $y + x\frac{dy}{dx}$ as derivative of xy	B1	
		State or imply $3y^2\frac{dy}{dx}\exp(1 + y^3)$ as derivative of $(1 + y^3)$	B1	
		Equate derivatives and solve for $\frac{dy}{dx}$	M1	
		Obtain the given answer	A1	[4]
		[The M1 is dependent on at least one of the B marks being earned]		
(ii)	Equate denominator to zero and solve for y		M1*	
	Obtain $y = 0.693$ only		A1	
	Substitute found value in the equation and solve for x		M1(dep*)	
	Obtain $x = 5.47$ only		A1	[4]

Differentiation 2 - Marking Scheme

Q6.

(i) Either	Use correct quotient rule or equivalent to obtain		
	$\frac{dx}{dt} = \frac{4(2t+3) - 8t}{(2t+3)^2}$ or equivalent	B1	
	Obtain $\frac{dy}{dt} = \frac{4}{2t+3}$ or equivalent	B1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$ or equivalent	M1	
	Obtain $\frac{1}{3}(2t+3)$ or similarly simplified equivalent	A1	
Or	Express t in terms of x or y e.g. $t = \frac{3x}{4-2x}$	B1	
	Obtain Cartesian equation e.g. $y = 2\ln\left(\frac{6}{2-x}\right)$	B1	
	Differentiate and obtain $\frac{dy}{dx} = \frac{2}{2-x}$	M1	
	Obtain $\frac{1}{3}(2t+3)$ or similarly simplified equivalent	A1	[4]
(ii)	Obtain $2t = 3$ or $t = \frac{3}{2}$	B1	
	Substitute in expression for $\frac{dy}{dx}$ and obtain 2	B1	[2]

Q7.

(i)	Use correct quotient rule or equivalent	M1	
	Obtain $\frac{(1+e^{2x})2x - (1+x^2)2e^{2x}}{(1+e^{2x})^2}$ or equivalent	A1	
	Substitute $x = 0$ and obtain $-\frac{1}{2}$ or equivalent	A1	[3]
(ii)	Differentiate y^3 and obtain $3y^2\frac{dy}{dx}$	B1	
	Differentiate $5xy$ and obtain $5y + 5x\frac{dy}{dx}$	B1	
	Obtain $6x^2 + 5y + 5x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$	B1	
	Substitute $x = 0, y = 2$ to obtain $-\frac{5}{6}$ or equivalent following correct work	B1	[4]

Differentiation 2 - Marking Scheme

Q8.

EITHER:	State $2ay\frac{dy}{dx}$ as derivative of ay^2	B1
	State $y^2 + 2xy\frac{dy}{dx}$ as derivative of xy^2	B1
	Equate derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero	M1
	Obtain $3x^2 + y^2 - 6ax = 0$, or horizontal equivalent	A1
	Eliminate y and obtain an equation in x	M1
	Solve for x and obtain answer $x = \sqrt{3}a$	A1
OR1:	Rearrange equation in the form $y^2 = \frac{3ax^2 - x^3}{x+a}$ and attempt differentiation of one side	B1
	Use correct quotient or product rule to differentiate RHS	M1
	Obtain correct derivative of RHS in any form	A1
	Set $\frac{dy}{dx}$ equal to zero and obtain an equation in x	M1
	Obtain a correct horizontal equation free of surds	A1
	Solve for x and obtain answer $x = \sqrt{3}a$	A1
OR2:	Rearrange equation in the form $y = \left(\frac{3ax^2 - x^3}{x+a}\right)^{\frac{1}{2}}$ and differentiation of RHS	B1
	Use correct quotient or product rule and chain rule	M1
	Obtain correct derivative in any form	A1

Equate derivative to zero and obtain an equation in x	M1	
Obtain a correct horizontal equation free of surds	A1	
Solve for x and obtain answer $x = \sqrt{3}a$	A1	[6]

Q9.

(i)	Use product rule	M1	
	Obtain correct derivative in any form, e.g. $4\sin 2x \cos 2x \cos x - \sin^2 2x \sin x$	A1	
	Equate derivative to zero and use a double angle formula	M1*	
	Reduce equation to one in a single trig function	M1(dep*)	
	Obtain a correct equation in any form, e.g. $10\cos^3 x = 6\cos x$, $4 = 6\tan^2 x$ or $4 = 10\sin^2 x$	A1	
	Solve and obtain $x = 0.685$	A1	[6]

Differentiation 2 - Marking Scheme

Q10.

Use correct quotient or product rule	M1	
Obtain correct derivative in any form	A1	
Justify the given statement	A1	**[3]**