Differentiation 2

Q1.

The equation of a curve is $3x^2 - 4xy + y^2 = 45$.

(i) Find the gradient of the curve at the point $(2, -3)$. [4]

(ii) Show that there are no points on the curve at which the gradient is 1. [3]

Q2.

The equation of a curve is $y = 3\sin x + 4\cos^3 x$.

(i) Find the x-coordinates of the stationary points of the curve in the interval $0 < x < \pi$. [6]

(ii) Determine the nature of the stationary point in this interval for which x is least. [2]

Q3.

The parametric equations of a curve are

$$x = \sin 2\theta - \theta, \quad y = \cos 2\theta + 2\sin\theta.$$

Show that $\dfrac{dy}{dx} = \dfrac{2\cos\theta}{1 + 2\sin\theta}$. [5]

Q4.

The curve with equation $y = \dfrac{e^{2x}}{x^3}$ has one stationary point.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

Q5.

The equation of a curve is $\ln(xy) - y^3 = 1$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{y}{x(3y^3 - 1)}$. [4]

(ii) Find the coordinates of the point where the tangent to the curve is parallel to the y-axis, giving each coordinate correct to 3 significant figures. [4]

Q6.

The parametric equations of a curve are

$$x = \frac{4t}{2t+3}, \quad y = 2\ln(2t+3).$$

(i) Express $\frac{dy}{dx}$ in terms of t, simplifying your answer. [4]

(ii) Find the gradient of the curve at the point for which $x = 1$. [2]

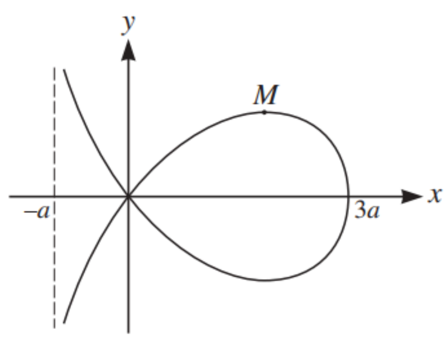
Q7.

For each of the following curves, find the gradient at the point where the curve crosses the y-axis:

(i) $y = \frac{1+x^2}{1+e^{2x}}$; [3]

(ii) $2x^3 + 5xy + y^3 = 8$. [4]

Q8.

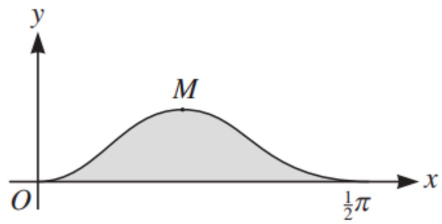


The diagram shows the curve with equation

$$x^3 + xy^2 + ay^2 - 3ax^2 = 0,$$

where a is a positive constant. The maximum point on the curve is M. Find the x-coordinate of M in terms of a. [6]

Q9.



The diagram shows the curve $y = \sin^2 2x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Find the x-coordinate of M. [6]

Q10.

The equation of a curve is $y = \dfrac{1+x}{1+2x}$ for $x > -\frac{1}{2}$. Show that the gradient of the curve is always negative. [3]
