

Logarithmic and Exponential Functions 1 MS

Q1.

- (i) *EITHER:* State or imply $n \ln x + \ln y = \ln C$ B1
 Substitute x - and y -values and solve for n M1
 Obtain $n = 1.50$ A1
 Solve for C M1
 Obtain $C = 6.00$ A1
- OR:* Obtain two correct equations by substituting x - and y -values in $x^n y = C$ B1
 Solve for n M1
 Obtain $n = 1.50$ A1
 Solve for C M1
 Obtain $C = 6.00$ A1 [5]
- (ii) State that the graph of $\ln y$ against $\ln x$ has equation $n \ln x + \ln y = \ln C$ which is *linear* in $\ln y$ and $\ln x$, or has equation of the form $nX + Y = \ln C$, where $X = \ln x$ and $Y = \ln y$, and is thus a straight line B1 [1]

Q2.

- EITHER:* Attempt to solve for 2^x M1
 Obtain $2^x = 6/4$, or equivalent A1
 Use correct method for solving an equation of the form $2^x = a$, where $a > 0$ M1
 Obtain answer $x = 0.585$ A1
- OR:* State an appropriate iterative formula, e.g. $x_{n+1} = \ln((2^{x_n} + 6) / 5) / \ln 2$ B1
 Use the iterative formula correctly at least once M1
 Obtain answer $x = 0.585$ A1
 Show that the equation has no other root but 0.585 A1 [4]

[For the solution 0.585 with no relevant working, award B1 and a further B1 if 0.585 is shown to be the only root.]

Q3.

- (i) State or imply $3 \ln y = \ln A + 2x$ at any stage B1
 State gradient is $\frac{2}{3}$, or equivalent B1 [2]
- (ii) Substitute $x = 0$, $\ln y = 0.5$ and solve for A M1
 Obtain $A = 4.48$ A1 [2]

Logarithmic and Exponential Functions 1 MS

Q4.

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|---|----|-----|
| Use law for the logarithm of a power, a quotient, or a product correctly at least once | M1 | |
| Use $\ln e = 1$ or $e = \exp(1)$ | M1 | |
| Obtain a correct equation free of logarithms, e.g. $1 + x^2 = ex^2$ | A1 | |
| Solve and obtain answer $x = 0.763$ only | A1 | [4] |
| [For the solution $x = 0.763$ with no relevant working give B1, and a further B1 if 0.763 is shown to be the only root.] | | |
| [Treat the use of logarithms to base 10 with answer 0.333 only, as a misread.] | | |
| [SR: Allow iteration, giving B1 for an appropriate formula, e.g. $x_{n+1} = \exp((\ln(1 + x_n^2) - 1)/2)$, M1 for using it correctly once, A1 for 0.763, and A1 for showing the equation has no other root but 0.763.] | | |

Q5.

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|--|----|-----|
| (i) Verify that $-96 + 100 + 8 - 12 = 0$ | B1 | |
| Attempt to find quadratic factor by division by $(x + 2)$, reaching a partial quotient $12x^2 + kx$, inspection or use of an identity | M1 | |
| Obtain $12x^2 + x - 6$ | A1 | |
| State $(x + 2)(4x + 3)(3x - 2)$ | A1 | [4] |
| [The M1 can be earned if inspection has unknown factor $Ax^2 + Bx - 6$ and an equation in A and/or B or equation $12x^2 + Bx + C$ and an equation in B and/or C .] | | |
| (ii) State $3^y = \frac{2}{3}$ and no other value | B1 | |
| Use correct method for finding y from equation of form $3^y = k$, where $k > 0$ | M1 | |
| Obtain -0.369 and no other value | A1 | [3] |

Q6.

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|---|----|-----|
| (i) Use at least one of $e^{2x} = 9$, $e^y = 2$ and $e^{2y} = 4$ | B1 | |
| Obtain given result $58 + 2k = c$ AG | B1 | [2] |
| (ii) Differentiate left-hand side term by term, reaching $ae^{2x} + be^y \frac{dy}{dx} + ce^{2y} \frac{dy}{dx}$ | M1 | |
| Obtain $12e^{2x} + ke^y \frac{dy}{dx} + 2e^{2y} \frac{dy}{dx}$ | A1 | |
| Substitute $(\ln 3, \ln 2)$ in an attempt involving implicit differentiation at least once, where | | |
| RHS = 0 | M1 | |
| Obtain $108 - 12k - 48 = 0$ or equivalent | A1 | |
| Obtain $k = 5$ and $c = 68$ | A1 | [5] |

Logarithmic and Exponential Functions 1 MS

Q7.

- (i) Use law for the logarithm of a product or quotient M1
Use $\log_2 32 = 5$ or $2^5 = 32$ M1
Obtain $x^2 + 5x - 32 = 0$, or horizontal equivalent A1 [3]
- (ii) Solve a 3-term quadratic equation M1
Obtain answer $x = 3.68$ only, or exact equivalent, e.g. $\frac{\sqrt{153} - 5}{2}$ A1 [2]

Q8.

- Use correct quotient or product rule M1
Obtain correct derivative in any form, e.g. $-\frac{3 \ln x}{x^4} + \frac{1}{x^4}$ A1
Equate derivative to zero and solve for x an equation of the form $\ln x = a$, where $a > 0$ M1
Obtain answer $\exp(\frac{1}{3})$, or 1.40, from correct work A1 [4]

Q9.

- Rearrange as $e^{2x} - e^x - 6 = 0$, or $u^2 - u - 6 = 0$, or equivalent B1
Solve a 3-term quadratic for e^x or for u M1
Obtain simplified solution $e^x = 3$ or $u = 3$ A1
Obtain final answer $x = 1.10$ and no other A1 [4]

Q10.

- State or imply $4 - 2^x = -10$ and 10 B1
Use correct method for solving equation of form $2^x = a$ M1
Obtain 3.81 A1 [3]