

Logarithmic and Exponential Functions 2 MS



Q1.

<i>EITHER:</i>	Use law of the logarithm of a power or quotient and remove logarithms	M1	
	Obtain a 3-term quadratic equation $x^2 - x - 3 = 0$, or equivalent	A1	
	Solve 3-term quadratic obtaining 1 or 2 roots	M1	
	Obtain answer 2.30 only	A1	
<i>OR1:</i>	Use an appropriate iterative formula, e.g. $x_{n+1} = \exp\left(\frac{1}{2} \ln(3x_n + 4)\right) - 1$ correctly at least once	M1	
	Obtain answer 2.30	A1	
	Show sufficient iterations to at least 3 d.p. to justify 2.30 to 2 d.p., or show there is a sign change in the interval (2.295, 2.305)	A1	
	Show there is no other root	A1	
<i>OR2:</i>	Use calculated values to obtain at least one interval containing the root	M1	
	Obtain answer 2.30	A1	
	Show sufficient calculations to justify 2.30 to 3 s.f., e.g. show it lies in (2.295, 2.305)	A1	
	Show there is no other root	A1	[4]

Q2.

	Use law of the logarithm of a power and a product or quotient and remove logarithms	M1	
	Obtain a correct equation in any form, e.g. $\frac{2x+3}{x^2} = 3$	A1	
	Solve 3-term quadratic obtaining at least one root	M1	
	Obtain final answer 1.39 only	A1	[4]

Q3.

<i>EITHER</i>	Use laws of indices correctly and solve for 5^x or for 5^{-x} or for 5^{x-1}	M1	
	Obtain 5^x or for 5^{-x} or for 5^{x-1} in any correct form, e.g. $5^x = \frac{5}{1 - 1/5}$	A1	
	Use correct method for solving $5^x = a$, or $5^{-x} = a$, or $5^{x-1} = a$, where $a > 0$	M1	
	Obtain answer $x = 1.14$	A1	
<i>OR</i>	Use an appropriate iterative formula, e.g. $x_{n+1} = \frac{\ln(5^{x-1} + 5)}{\ln 5}$, correctly, at least once	M1	
	Obtain answer 1.14	A1	
	Show sufficient iterations to at least 3 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145)	A1	
	Show there is no other root	A1	[4]
	[For the solution $x = 1.14$ with no relevant working give B1, and a further B1 if 1.14 is shown to be the only solution.]		

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Q4.

- State or imply $\ln e = 1$ B1
 Apply at least one logarithm law for product or quotient correctly
 (or exponential equivalent) M1
 Obtain $x + 5 = ex$ or equivalent and hence $\frac{5}{e-1}$ A1 [3]

Q5.

- (i) Either State or imply non-modular equation $(4x-1)^2 = (x-3)^2$ or pair of
 linear equations $4x-1 = \pm(x-3)$ B1
 Solve a three-term quadratic equation or two linear equations M1
 Obtain $-\frac{2}{3}$ and $\frac{4}{5}$ A1
- Or Obtain value $-\frac{2}{3}$ from inspection or solving linear equation B1
 Obtain value $\frac{4}{5}$ similarly B2 [3]
- (ii) State or imply at least $4^y = \frac{4}{5}$, following a positive answer from part (i) B1√
 Apply logarithms and use $\log a^b = b \log a$ property M1
 Obtain -0.161 and no other answer A1 [3]

Q6.

- EITHER:* State or imply $\ln y = \ln A - kx^2$ B1
 Substitute values of $\ln y$ and x^2 , and solve for k or $\ln A$ M1
 Obtain $k = 0.42$ or $A = 2.80$ A1
 Solve for $\ln A$ or k M1
 Obtain $A = 2.80$ or $k = 0.42$ A1
- OR1:* State or imply $\ln y = \ln A - kx^2$ B1
 Using values of $\ln y$ and x^2 , equate gradient of line to $-k$ and solve for k M1
 Obtain $k = 0.42$ A1
 Solve for $\ln A$ M1
 Obtain $A = 2.80$ A1
- OR2:* Obtain two correct equations in k and A and substituting y - and x^2 - values in
 $y = Ae^{-kx^2}$ B1
 Solve for k M1
 Obtain $k = 0.42$ A1
 Solve for A M1
 Obtain $A = 2.80$ A1 [5]
 [SR: If unsound substitutions are made, e.g. using $x = 0.64$ and $y = 0.76$, give
 B1M0A0M1A0 in the *EITHER* and *OR1* schemes, and B0M1A0M1A0 in the *OR2*
 scheme.]

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Q7.

- Use law for the logarithm of a product, quotient or power M1
 Use $\ln e = 1$ or $\exp(1) = e$ M1
 Obtain correct equation free of logarithms in any form, e.g. $\frac{y+1}{y} = ex^3$ A1
 Rearrange as $y = (ex^3 - 1)^{-1}$, or equivalent A1 [4]

Q8.

- EITHER:* State or imply non-modular equation $2^2(3^x - 1)^2 = (3^x)^2$, or pair of equations
 $2(3^x - 1) = \pm 3^x$ M1
 Obtain $3^x = 2$ and $3^x = \frac{2}{3}$ (or $3^{x+1} = 2$) A1
OR: Obtain $3^x = 2$ by solving an equation or by inspection B1
 Obtain $3^x = \frac{2}{3}$ (or $3^{x+1} = 2$) by solving an equation or by inspection B1
 Use correct method for solving an equation of the form $3^x = a$ (or $3^{x+1} = a$), where $a > 0$ M1
 Obtain final answers 0.631 and -0.369 A1 [4]

Q9.

- Apply at least one logarithm property correctly *M1
 Obtain $\frac{(x+4)^2}{x} = x+a$ or equivalent **without logarithm** involved A1
 Rearrange to express x in terms of a M1 d*M
 Obtain $\frac{16}{a-8}$ or equivalent A1 [4]

Q10.

- (i) Use law for the logarithm for a product or quotient or exponentiation AND for a power M1
 Obtain $(4x - 5)^2(x + 1) = 27$ B1
 Obtain given equation correctly $16x^3 - 24x^2 - 15x - 2 = 0$ A1 [3]
- (ii) Obtain $x = 2$ is root or $(x - 2)$ is a factor, or likewise with $x = -\frac{1}{4}$ B1
 Divide by $(x - 2)$ to reach a quotient of the form $16x^2 + kx$ M1
 Obtain quotient $16x^2 + 8x + 1$ A1
 Obtain $(x - 2)(4x + 1)^2$ or $(x - 2), (4x + 1), (4x + 1)$ A1 [4]
- (iii) State $x = 2$ only A1 [1]