

Numerical solutions of equations 1 MS



Q1.

- (i) Using the formulae $\frac{1}{2}r^2\theta$ and $\frac{1}{2}r^2 \sin \theta$, or equivalent, form an equation M1
 Obtain a correct equation in r and x and/or $x/2$ in any form A1
 Obtain the given equation correctly A1 [3]
- (ii) Consider the sign of $x - (\frac{3}{4}\pi - \sin x)$ at $x = 1.3$ and $x = 1.5$, or equivalent M1
 Complete the argument with correct calculations A1 [2]
- (iii) Use the iterative formula correctly at least once M1
 Obtain final answer 1.38 A1
 Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.375, 1.385) A1 [3]

Q2.

- (i) Use correct quotient or product rule M1
 Obtain correct derivative in any form A1
 Equate derivative to zero and solve for x M1
 Obtain the given answer correctly A1 [4]
- (ii) Use the iterative formula correctly at least once M1
 Obtain final answer 4.49 A1
 Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show that there is a sign change in the interval (4.485, 4.495) A1 [3]

Q3.

- (i) Use correct quotient or product rule M1
 Obtain correct derivative in any form, e.g. $\frac{1}{x(x+1)} - \frac{\ln x}{(x+1)^2}$ A1
 Equate derivative to zero and obtain the given equation correctly A1
 Consider the sign of $x - \frac{(x+1)}{\ln x}$ at $x = 3$ and $x = 4$, or equivalent M1
 Complete the argument with correct calculated values A1 [5]
- (ii) Use the iterative formula correctly at least once, using or reaching a value in the interval (3, 4) M1
 Obtain final answer 3.59 A1
 Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (3.585, 3.595) A1 [3]

Q4.

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|---|-----------------------------|
| <p>(i) Make recognisable sketch of a relevant graph over the given range
Sketch the other relevant graph on the same diagram and justify the given statement</p> | <p>B1
B1 [2]</p> |
| <p>(ii) Consider sign of $4x^2 - 1 - \cot x$ at $x = 0.6$ and $x = 1$, or equivalent
Complete the argument correctly with correct calculated values</p> | <p>M1
A1 [2]</p> |
| <p>(iii) Use the iterative formula correctly at least once
Obtain final answer 0.73
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.725, 0.735)</p> | <p>M1
A1
A1 [3]</p> |

Q5.

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|--|---|
| <p>(i) Attempt integration by parts
Obtain $-x^{-1} \ln x + \int \frac{1}{x^2} dx$, $\frac{x \ln x - x}{x^2} + 2 \int \frac{\ln x}{x^2} dx - 2 \int \frac{1}{x^2} dx$ or equivalent
Obtain $-x^{-1} \ln x - x^{-1}$ or equivalent
Use limits correctly, equate to $\frac{2}{3}$ and attempt rearrangement to obtain a in terms of $\ln a$
Obtain given answer $a = \frac{5}{3}(1 + \ln a)$ correctly</p> | <p>M1
A1
A1
M1
A1 [5]</p> |
| <p>(ii) Use valid iterative formula correctly at least once
Obtain final answer 3.96
Show sufficient iterations to > 4 dp to justify accuracy to 2 dp or show sign change in interval (3.955, 3.965)
[4 → 3.9772 → 3.9676 → 3.9636 → 3.9619]

SR: Use of $a_{n+1} = e^{\left(\frac{2}{3}a_n - 1\right)}$ to obtain 0.50 also earns 3/3.</p> | <p>M1
A1
A1 [3]</p> |

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Q6.

- (i) State or imply area of segment is $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ or $50\theta - 50\sin\theta$ B1
 Attempt to form equation from area of segment = $\frac{1}{5}$ of area of circle, or equivalent M1
 Confirm given result $\theta = \frac{2}{5}\pi + \sin\theta$ A1 [3]
- (ii) Use iterative formula correctly at least once M1
 Obtain value for θ of 2.11 A1
 Show sufficient iterations to justify value of θ or show sign change in interval (2.105, 2.115) A1
 Use correct trigonometry to find an expression for the length of AB M1
 e.g. $20\sin 1.055$ or $\sqrt{200 - 200\cos 2.11}$
 Hence 17.4 A1 [5]
 [2.1 \rightarrow 2.1198 \rightarrow 2.1097 \rightarrow 2.1149 \rightarrow 2.1122]

Q7.

- (i) State or imply $CT = r \tan x$ or $OT = r \sec x$, or equivalent B1
 Using correct area formulae, form an equation in r and x M1
 Obtain the given answer correctly A1 [3]
- (ii) Use the iterative formula correctly at least once M1
 Obtain the final answer 1.35 A1
 Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.345, 1.355) A1 [3]

Q8.

- (i) Make recognisable sketch of a relevant graph over the given range B1
 Sketch the other relevant graph and justify the given statement B1 [2]
- (ii) Consider the sign of $\cot x - (1 + x^2)$ at $x = 0.5$ and $x = 0.8$, or equivalent M1
 Complete the argument with correct calculated values A1 [2]
- (iii) Use the iterative formula correctly at least once with $0.5 \leq x_n \leq 0.8$ M1
 Obtain final answer 0.62 A1
 Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.615, 0.625) A1 [3]

