Numerical solutions of equations 2 MS

Q1.

(i)	Use correct identity for $\tan 2x$ and obtains $at^4 + bt^3 + ct^2 + dt = 0$, where b may be zero	M1	
	Obtain correct horizontal equation, e.g. $4t + 5t^2 - 5t^4 = 0$	A1	
	Obtain $kt(t^3 + et + f) = 0$ or equivalent	M1	
	Confirm given results $t = 0$ and $t = \sqrt[3]{t + 0.8}$	A1	[4]
(ii)	Consider sign of $t - \sqrt[3]{t + 0.8}$ at 1.2 and 1.3 or equivalent	M1	
	Justify the given statement with correct calculations (–0.06 and 0.02)	A1	[2]
(iii)	Use the iterative formula correctly at least once with $1.2 < t_n < 1.3$	M1	
	Obtain final answer 1.276	A1	
	Show sufficient iterations to justify answer or show there is a change of sign in interval (1.2755, 1.2765)	A1	[3]
(iv)	Evaluate $\tan^{-1}$ (answer from part **(iii)**) to obtain at least one value	M1	
	Obtain –2.24 and 0.906	A1	
	State $-\pi$, 0 and π	B1	[3]
	[SR If A0, B0, allow B1 for any 3 roots]		

Q2.

(i)	Using the formulae $\frac{1}{2}r^2\theta$ and $\frac{1}{2}bh$, form an equation an a and θ	M1	
	Obtain given answer	A1	[2]
(ii)	Use the iterative formula correctly at least once	M1	
	Obtain answer $\theta = 1.32$	A1	
	Show sufficient iterations to 4 d.p. to justify 1.32 to 2 d.p., or show there is a sign change in the interval (1.315, 1.325)	A1	[3]

Q3.

(i)	Use correct product or quotient rule and use chain rule at least once	M1	
	Obtain derivative in any correct form	A1	
	Equate derivative to zero and solve an equation with at least two non-zero terms for real x	M1	
	Obtain answer $x = \frac{1}{\sqrt{2}}$, or exact equivalent	A1	[4]
(ii)	State a suitable equation, e.g. $\alpha = \sqrt{(\ln(4 + 8\alpha^2))}$	B1	
	Rearrange to reach $e^{\alpha^2} = 4 + 8\alpha^2$	B1	
	Obtain $\frac{1}{2} = e^{-\frac{1}{2}\alpha^2}\sqrt{(1 + 2\alpha^2)}$, or work *vice versa*	B1	[3]
(iii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.86	A1	
	Show sufficient iterations to 4 d.p. to justify 1.86 to 2 d.p., or show there is a sign change in the interval (1.855, 1.865)	A1	[3]

Q4.

(i)	Find y for $x = -2$		M1	
	Obtain 0 and conclude that $\alpha = -2$		A1	[2]
(ii)	Either	Find cubic factor by division or inspection or equivalent	M1	
		Obtain $x^3 + 2x - 8$	A1	
		Rearrange to confirm given equation $x = \sqrt[3]{8 - 2x}$	A1	
	Or	Derive cubic factor from given equation and form product with $(x - \alpha)$	M1	
		$(x+2)(x^3 + 2x - 8)$	A1	
		Obtain quartic $x^4 + 2x^3 + 2x^2 - 4x - 16\ (= 0)$	A1	
	Or	Derive cubic factor from given equation and divide the quartic by the cubic	M1	
		$(x^4 + 2x^3 + 2x^2 - 4x - 16) \div (x^3 + 2x - 8)$	A1	
		Obtain correct quotient and zero remainder	A1	[3]
(iii)	Use the given iterative formula correctly at least once		M1	
	Obtain final answer 1.67		A1	
	Show sufficient iterations to at least 4 d.p. to justify answer 1.67 to 2 d.p. or show there is a change of sign in interval (1.665, 1.675)		A1	[3]

Q5.

(i)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 3.6840	A1	
	Show sufficient iterations to at least 6 d.p. to justify 3.6840, or show there is a sign change in the interval (3.68395, 3.68405)	A1	[3]
(ii)	State a suitable equation, e.g. $x = \dfrac{x(x^3+100)}{2(x^3+25)}$	B1	
	State that the value of α is $3\sqrt{50}$, or exact equivalent	B1	[2]

Q6.

(i)	State the correct derivatives $2e^{2x-3}$ and $2/x$	B1	
	Equate derivatives and use a law of logarithms on an equation equivalent to $ke^{2x-3} = m/x$	M1	
	Obtain the given result correctly (or work *vice versa*)	A1	[3]
(ii)	Consider the sign of $a - \frac{1}{2}(3 - \ln a)$ when $a = 1$ and $a = 2$, or equivalent	M1	
	Complete the argument with correct calculated values	A1	[2]
(iii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.35	A1	
	Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p., or show there is a sign change in the interval (1.345, 1.355)	A1	[3]

Q7.

(i)	State or imply $AB = 2r\cos\theta$ or $AB^2 = 2r^2 - 2r^2\cos(\pi - 2\theta)$	B1	
	Use correct formula to express the area of sector ABC in terms of r and θ	M1	
	Use correct area formulae to express the area of a segment in terms of r and θ	M1	
	State a correct equation in r and θ in any form	A1	
	Obtain the given answer	A1	**[5]**
	[SR: If the complete equation is approached by adding two sectors to the shaded area above BO and OC give the first M1 as on the scheme, and the second M1 for using correct area formulae for a triangle AOB or AOC, and a sector AOB or AOC.]		
(ii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 0.95	A1	
	Show sufficient iterations to 4 d.p. to justify 0.95 to 2 d.p., or show there is a sign change in the interval (0.945, 0.955)	A1	**[3]**

Q8.

(i)	Use integration by parts to obtain $axe^{-\frac{1}{2}x} + \int be^{-\frac{1}{2}x}\,dx$	M1*	
	Obtain $-8xe^{-\frac{1}{2}x} + \int 8e^{-\frac{1}{2}x}\,dx$ or unsimplified equivalent	A1	
	Obtain $-8xe^{-\frac{1}{2}x} - 16e^{-\frac{1}{2}x}$	A1	
	Use limits correctly and equate to 9	M1(d*M)	
	Obtain given answer $p = 2\ln\left(\frac{8p+16}{7}\right)$ correctly	A1	[5]
(ii)	Use correct iteration formula correctly at least once	M1	
	Obtain final answer 3.77	A1	
	Show sufficient iterations to 5sf or better to justify accuracy 3.77 or show sign change in interval (3.765, 3.775)	A1	[3]
	$[3.5 \to 3.6766 \to 3.7398 \to 3.7619 \to 3.7696 \to 3.7723]$		

Q9.

(i)	Sketch $y = \text{cosec}\,x$ for at least $0, x, \pi$	B1	
	Sketch $y = x(\pi - x)$ for at least $0, x, \pi$	B1	
	Justify statement concerning two roots, with evidence of 1 and $\frac{1}{4}\pi^2$ for y-values on graph via scales	B1	[3]
(ii)	Use $\text{cosec}x = \frac{1}{\sin x}$ and commence rearrangement	M1	
	Obtain given equation correctly, showing sufficient detail	A1	[2]
(iii)	**(a)** Use the iterative formula correctly at least once	M1	
	Obtain final answer 0.66	A1	
	Show sufficient iterations to 4 decimal places to justify answer or show a sign change in the interval $(0.655, 0.665)$	A1	[3]
	(b) Obtain 2.48	B1	[1]

Q10.

(i)	Use correct arc formula and form an equation in r and x	M1	
	Obtain a correct equation in any form	A1	
	Rearrange in the given form	A1	**3**
(ii)	Consider sign of a relevant expression at $x = 1$ and $x = 1.5$, or compare values of relevant expressions at $x = 1$ and $x = 1.5$	M1	
	Complete the argument correctly with correct calculated values	A1	**2**
(iii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.21	A1	
	Show sufficient iterations to 4 d.p. to justify 1.21 to 2 d.p., or show there is a sign change in the interval (1.205,1.215)	A1	**3**