Complex Numbers 1 MS

Q1.

(i)	Obtain modulus $\sqrt{8}$	B1	
	Obtain argument $\frac{1}{4}\pi$ or 45°	B1	[2]
(ii)	Show 1, i and u in relatively correct positions on an Argand diagram	B1	
	Show the perpendicular bisector of the line joining 1 and i	B1	
	Show a circle with centre u and radius 1	B1	
	Shade the correct region	B1	[4]
(iii)	State or imply relevance of the appropriate tangent from O to the circle	B1 √	
	Carry out complete strategy for finding $\lvert z \rvert$ for the critical point	M1	
	Obtain answer $\sqrt{7}$	A1	[3]

Q2.

(i)	*EITHER*:	State a correct expression for $\lvert z\rvert$ or $\lvert z\rvert^2$, e.g. $(1+\cos 2\theta)^2 + (\sin 2\theta)^2$	B1	
		Use double angle formulae throughout or Pythagoras	M1	
		Obtain given answer 2cos θ correctly	A1	
		State a correct expression for tangent of argument, e.g. $(\sin 2\theta/(1+\cos 2\theta)$	B1	
		Use double angle formulae to express it in terms of cos θ and sin θ	M1	
		Obtain tan θ and state that the argument is θ	A1	
	OR:	Use double angle formulae to express z in terms of cos θ and sin θ	M1	
		Obtain a correct expression, e.g. $1+\cos^2\theta - \sin^2\theta + 2\mathrm{i}\sin\theta\cos\theta$	A1	
		Convert the expression to polar form	M1	
		Obtain $2\cos\theta(\cos\theta + \mathrm{i}\sin\theta)$	A1	
		State that the modulus is 2 cosθ	A1	
		State that the argument is θ	A1	[6]
(ii)		Substitute for z and multiply numerator and denominator by the conjugate of z, or equivalent	M1	
		Obtain correct real denominator in any form	A1	
		Identify and obtain real part equal to $\frac{1}{2}$	A1	[3]

Q3.

(a)	*EITHER*:	Substitute $1+i\sqrt{3}$, attempt complete expansions of the x^3 and x^2 terms	M1	
		Use $i^2 = -1$ correctly at least once	B1	
		Complete the verification correctly	A1	
		State that the other root is $1-i\sqrt{3}$	B1	
	*OR*1:	State that the other root is $1-i\sqrt{3}$	B1	
		State quadratic factor $x^2 - 2x + 4$	B1	
		Divide cubic by 3-term quadratic reaching partial quotient $2x + k$	M1	
		Complete the division obtaining zero remainder	A1	
	*OR*2:	State factorisation $(2x+3)(x^2 - 2x + 4)$, or equivalent	B1	
		Make reasonable solution attempt at a 3-term quadratic and use $i^2 = -1$	M1	
		Obtain the root $1+i\sqrt{3}$	A1	
		State that the other root is $1-i\sqrt{3}$	B1	[4]

(b)	Show point representing $1+i\sqrt{3}$ in relatively correct position on an Argand diagram	B1	
	Show circle with centre at $1+i\sqrt{3}$ and radius 1	B1√	
	Show line for $\arg z = \frac{1}{3}\pi$ making $\frac{1}{3}\pi$ with the real axis	B1	
	Show line from origin passing through centre of circle, or the diameter which would contain the origin if produced	B1	
	Shade the relevant region	B1√	[5]

Q4.

(i)	State modulus is 2	B1	
	State argument is $\frac{1}{6}\pi$, or 30°, or 0.524 radians	B1	[2]

(ii)	**(a)**		State answer $3\sqrt{3}+i$	B1	
	(b)	*EITHER*:	Multiply numerator and denominator by $\sqrt{3}-i$, or equivalent	M1	
			Simplify denominator to 4 or numerator to $2\sqrt{3}+2i$	A1	
			Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}i$, or equivalent	A1	
		OR 1:	Obtain two equations in x and y and solve for x or for y	M1	
			Obtain $x = \frac{1}{2}\sqrt{3}$ or $y = \frac{1}{2}$	A1	
			Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}i$, or equivalent	A1	
		OR 2:	Using the correct processes express iz^*/z in polar form	M1	
			Obtain $x = \frac{1}{2}\sqrt{3}$ or $y = \frac{1}{2}$	A1	
			Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}i$, or equivalent	A1	[4]

(iii)	Plot A and B in relatively correct positions	B1	
	EITHER: Use fact that angle $AOB = \arg(\mathrm{i}z^*) - \arg z$	M1	
	Obtain the given answer	A1	
	OR 1: Obtain tan $A\hat{O}B$ from gradients of OA and OB and the correct $\tan(A-B)$ formula	M1	
	Obtain the given answer	A1	
	OR 2: Obtain cos $A\hat{O}B$ by using correct cosine formula or scalar product	M1	
	Obtain the given answer	A1	[3]

Q5.

(i)	Attempt multiplication and use $\mathrm{i}^2 = -1$	M1	
	Obtain $3 + 4\mathrm{i}$	A1	
	Obtain 5 for modulus	B1	[3]
(ii)	Draw complete circle with centre corresponding to their w^2 …	B1√	
	… and radius corresponding to their $\lvert w^2 \rvert$	B1√	
	Shade the correct region	cwo B1	[3]

Q6.

(i)	Either: Multiply numerator and denominator by $(1 - 2\mathrm{i})$, or equivalent	M1	
	Obtain $-3\mathrm{i}$	A1	
	State modulus is 3	A1	
	Refer to u being on negative imaginary axis or equivalent and confirm argument as $-\frac{1}{2}\pi$	A1	
	Or: Using correct processes, divide moduli of numerator and denominator	M1	
	Obtain 3	A1	
	Subtract argument of denominator from argument of numerator	M1	
	Obtain $-\tan^{-1}\frac{1}{2} - \tan^{-1}2$ or $-0.464 - 1.107$ and hence $-\frac{1}{2}\pi$ or -1.57	A1	[4]
(ii)	Show correct half-line from u at angle $\frac{1}{4}\pi$ to real direction	B1	
	Use correct trigonometry to find required value	M1	
	Obtain $\frac{3}{2}\sqrt{2}$ or equivalent	A1	[3]
(iii)	Show, or imply, locus is a circle with centre $(1 + \mathrm{i})u$ and radius 1	M1	
	Use correct method to find distance from origin to furthest point of circle	M1	
	Obtain $3\sqrt{2} + 1$ or equivalent	A1	[3]

Q7.

(a) **(i)** *EITHER*: Multiply numerator and denominator by $a - 2\mathrm{i}$, or equivalent — M1

Obtain final answer $\frac{5a}{a^2+4} - \frac{10\mathrm{i}}{a^2+4}$, or equivalent — A1

OR: Obtain two equations in x and y, solve for x or for y — M1

Obtain final answer $x = \frac{5a}{a^2+4}$ and $y = \frac{10}{a^2+4}$, or equivalent — A1 [2]

(ii) Either state $\arg(u) = -\frac{3}{4}\pi$, or express u^* in terms of a (f.t. on u) — B1√

Use correct method to form an equation in a, e.g. $5a = -10$ — M1

Obtain $a = -2$ correctly — A1 [3]

(b) Show a point representing $2 + 2\mathrm{i}$ in relatively correct position in an Argand diagram — B1

Show the circle with centre at the origin and radius 2 — B1

Show the perpendicular bisector of the line segment from the origin to the point representing $2 + 2\mathrm{i}$ — B1√

Shade the correct region — B1 [4]

[SR: Give the first B1 and the B1√ for obtaining $y = 2 - x$, or equivalent, and sketching the attempt.]

Q8.

(i) Use the quadratic formula, completing the square, or the substitution $z = x + \mathrm{i}y$ to find a root and use $\mathrm{i}^2 = -1$ — M1

Obtain final answers $-\sqrt{3} \pm \mathrm{i}$, or equivalent — A1 [2]

(ii) State that the modulus of both roots is 2 — B1√

State that the argument of $-\sqrt{3} + \mathrm{i}$ is 150° or $\frac{5}{6}\pi$ (2.62) radians — B1√

State that the argument of $-\sqrt{3} - \mathrm{i}$ is –150° (or 210°) or $-\frac{5}{6}\pi$ (–2.62) radians or $\frac{7}{6}\pi$ (3.67) radians — B1√ [3]

(iii) Carry out an attempt to find the sixth power of a root — M1

Verify that one of the roots satisfies $z^6 = -64$ — A1

Verify that the other root satisfies the equation — A1 [3]

Q9.

(a) *EITHER*: Square $x + \mathrm{i}y$ and equate real and imaginary parts to 1 and $-2\sqrt{6}$ respectively M1*

Obtain $x^2 - y^2 = 1$ and $2xy = -2\sqrt{6}$ A1

Eliminate one variable and find an equation in the other M1(dep*)

Obtain $x^4 - x^2 - 6 = 0$ or $y^4 + y^2 - 6 = 0$, or 3-term equivalent A1

Obtain answers $\pm\ (\sqrt{3} - \mathrm{i}\sqrt{2})$ A1 [5]

OR: Denoting $1 - 2\sqrt{6}\mathrm{i}$ by $R\mathrm{cis}\theta$, state, or imply, square roots are $\pm\sqrt{R}\mathrm{cis}(\frac{1}{2}\theta)$ and find values of R and either $\cos\theta$ or $\sin\theta$ or $\tan\theta$ M1*

Obtain $\pm\sqrt{5}(\cos\frac{1}{2}\theta + \mathrm{i}\sin\frac{1}{2}\theta)$, and $\cos\theta = \frac{1}{5}$ or $\sin\theta = -\frac{2\sqrt{6}}{5}$ or $\tan\theta = -2\sqrt{6}$ A1

Use correct method to find an exact value of $\cos\frac{1}{2}\theta$ or $\sin\frac{1}{2}\theta$ M1(dep*)

Obtain $\cos\frac{1}{2}\theta = \pm\sqrt{\frac{3}{5}}$ and $\sin\frac{1}{2}\theta = \pm\sqrt{\frac{2}{5}}$, or equivalent A1

Obtain answers $\pm(\sqrt{3} - \mathrm{i}\sqrt{2})$, or equivalent A1

[Condone omission of $\pm$ except in the final answers.]

(b) Show point representing 3i on a sketch of an Argand diagram B1

Show a circle with centre at the point representing 3i and radius 2 B1√

Shade the interior of the circle B1√

Carry out a complete method for finding the greatest value of arg z M1

Obtain answer 131.8° or 2.30 (or 2.3) radians A1 [5]

[The f.t. is on solutions where the centre is at the point representing –3i.]

Q10.

6	**(i)**	Use correct method for finding modulus of their w^2 or w^3 or both	M1	
		Obtain $\lvert w^2 \rvert = 2$ and $\lvert w^3 \rvert = 2\sqrt{2}$ or equivalent	A1	
		Use correct method for finding argument of their w^2 or w^3 or both	M1	
		Obtain $\arg(w^2) = -\frac{1}{2}\pi$ or $\frac{3}{2}\pi$ and $\arg(w^3) = \frac{1}{4}\pi$	A1ft	[4]
	(ii)	Obtain centre $-\frac{1}{2} - \frac{1}{2}\mathrm{i}$ (their w^2)	B1ft	
		Calculate the diameter or radius using $\lvert w - w^2 \rvert$ w21 or right-angled triangle or cosine rule or equivalent	M1	
		Obtain radius $\frac{1}{2}\sqrt{10}$ or equivalent	A1	
		Obtain $\lvert z + \frac{1}{2} + \frac{1}{2}\mathrm{i} \rvert = \frac{1}{2}\sqrt{10}$ or equivalent	A1ft	[4]