Complex Numbers 2 MS

Q1.

(i)	Either	Expand $(1 + 2i)^2$ to obtain $-3 + 4i$ or unsimplified equivalent	B1	
		Multiply numerator and denominator by $2 - i$	M1	
		Obtain correct numerator $-2 + 11i$ or correct denominator 5	A1	
		Obtain $-\frac{2}{5}+\frac{11}{5}i$ or equivalent	A1	
	Or	Expand $(1 + 2i)^2$ to obtain $-3 + 4i$ or unsimplified equivalent	B1	
		Obtain two equations in x and y and solve for x or y	M1	
		Obtain final answer $x=-\frac{2}{5}$	A1	
		Obtain final answer $y=\frac{11}{5}$	A1	[4]
(ii)	Draw a circle		M1	
	Show centre at relatively correct position, following their u		A1√	
	Draw circle passing through the origin		A1	[3]

Q2.

(i)	*EITHER*:	Multiply numerator and denominator by $1 + 3i$, or equivalent	M1	
		Simplify numerator to $-5 + 5i$, or denominator to 10, or equivalent	A1	
		Obtain final answer $-\frac{1}{2}+\frac{1}{2}i$, or equivalent	A1	
	OR:	Obtain two equations in x and y, and solve for x or for y	M1	
		Obtain $x=-\frac{1}{2}$ or $y=\frac{1}{2}$, or equivalent	A1	
		Obtain final answer $-\frac{1}{2}+\frac{1}{2}i$, or equivalent	A1	[3]
(ii)	Show B and C in relatively correct positions in an Argand diagram		B1	
	Show u in a relatively correct position		B1√	[2]
(iii)	Substitute exact arguments in the LHS $\arg(1 + 2i) - \arg(1 - 3i) = \arg u$, or equivalent		M1	
	Obtain and use $\arg u=\frac{3}{4}\pi$		A1	
	Obtain the given result correctly		A1	[3]

Q3.

(a)	EITHER:	Eliminate u or w and obtain an equation in w or in u	M1	
		Obtain a quadratic in u or w, e.g. $u^2 - 4iu - 5 = 0$ or $w^2 + 4iw - 5 = 0$	A1	
		Solve a 3-term quadratic for u or for w	M1	
	OR1:	Having squared the first equation, eliminate u or w and obtain an equation in w or u	M1	
		Obtain a 2-term quadratic in u or w, e.g. $u^2 = -3 + 4i$	A1	
		Solve a 2-term quadratic for u or for w	M1	
	OR2:	Using $u = a + ib$, $w = c + id$, equate real and imaginary parts and obtain 4 equations in a, b, c and d	M1	
		Obtain 4 correct equations	A1	
		Solve for a and b, or for c and d	M1	
		Obtain answer $u = 1 + 2i$, $w = 1 - 2i$	A1	
		Obtain answer $u = -1 + 2i$, $w = -1 - 2i$ and no other	A1	[5]
(b)	**(i)**	Show point representing $2 - 2i$ in relatively correct position	B1	
		Show a circle with centre $2 - 2i$ and radius 2	B1√	
		Show line for $\arg z = -\frac{1}{4}\pi$	B1	
		Show line for $\text{Re } z = 1$	B1	
		Shade the relevant region	B1	[5]
	(ii)	State answer $2 + \sqrt{2}$, or equivalent (accept 3.41)	B1	[1]

Q4.

(i)	*EITHER*	Substitute $x = 1 + \sqrt{2}\,i$ and attempt the expansions of the x^2 and x^4 terms	M1	
		Use $i^2 = -1$ correctly at least once	B1	
		Complete the verification	A1	
		State second root $1 - \sqrt{2}\,i$	B1	
	OR 1	State second root $1 - \sqrt{2}\,i$	B1	
		Carry out a complete method for finding a quadratic factor with zeros $1 \pm \sqrt{2}\,i$	M1	
		Obtain $x^2 - 2x + 3$, or equivalent	A1	
		Show that the division of $p(x)$ by $x^2 - 2x + 3$ gives zero remainder and complete the verification	A1	
	OR 2	Substitute $x = 1 + \sqrt{2}\,i$ and use correct method to express x^2 and x^4 in polar form	M1	
		Obtain x^2 and x^4 in any correct polar form (allow decimals here)	B1	
		Complete an exact verification	A1	
		State second root $1 - \sqrt{2}\,i$, or its polar equivalent (allow decimals here)	B1	[4]

(ii)	Carry out a complete method for finding a quadratic factor with zeros $1 \pm \sqrt{2}\,i$	M1*
	Obtain $x^2 - 2x + 3$, or equivalent	A1
	Attempt division of p(x) by $x^2 - 2x + 3$ reaching a partial quotient $x^2 + kx$, or equivalent	M1 (dep*)
	Obtain quadratic factor $x^2 - 2x + 2$	A1
	Find the zeros of the second quadratic factor, using $i^2 = -1$	M1 (dep*)
	Obtain roots $-1 + i$ and $-1 - i$	A1 [6]

[The second M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + (6/3)$ and an equation in A and/or B]
[If part **(i)** is attempted by the *OR 1* method, then an attempt at part **(ii)** which uses or quotes relevant working or results obtained in part **(i)** should be marked using the scheme for part **(ii)**]

Q5.

(a)	Expand and simplify as far as $iw^2 = -8i$ or equivalent	B1
	Obtain first answer $i\sqrt{8}$, or equivalent	B1
	Obtain second answer $-i\sqrt{8}$, or equivalent and no others	B1 [3]
(b) (i)	Draw circle with centre in first quadrant	M1
	Draw correct circle with interior shaded or indicated	A1 [2]
(ii)	Identify ends of diameter corresponding to line through origin and centre	M1
	Obtain $p = 3.66$ and $q = 7.66$	A1
	Show tangents from origin to circle	M1
	Evaluate $\sin^{-1}\left(\frac{1}{4}\sqrt{2}\right)$	M1
	Obtain $\alpha = \frac{1}{4}\pi - \sin^{-1}\left(\frac{1}{4}\sqrt{2}\right)$ or equivalent and hence 0.424	A1
	Obtain $\beta = \frac{1}{4}\pi + \sin^{-1}\left(\frac{1}{4}\sqrt{2}\right)$ or equivalent and hence 1.15	A1 [6]

Q6.

(a)	State or imply $3a + 3bi + 2i(a - bi) = 17 + 8i$	B1
	Consider real and imaginary parts to obtain two linear equations in a and b	M1*
	Solve two simultaneous linear equations for a or b	M1 (dep*)
	Obtain $7 - 2i$	A1 [4]
(b) Either	Show or imply a triangle with side 2	B1
	State at least two of the angles $\frac{1}{4}\pi, \frac{2}{3}\pi$ and $\frac{1}{12}\pi$	B1
	State or imply argument is $\frac{1}{4}\pi$	B1
	Use sine rule or equivalent to find r	M1
	Obtain $6.69e^{\frac{1}{4}\pi i}$	A1

<u>Or</u>	State $y = x$.	B1	
	State $y = \frac{1}{\sqrt{3}}x + 2$ or $\frac{\sqrt{3}}{2} = \frac{x}{\sqrt{x^2 + (y-2)^2}}$ or $\frac{1}{2} = \frac{y-2}{\sqrt{x^2 + (y-2)^2}}$	B1	
	State or imply argument is $\frac{\pi}{4}$	B1	
	Solve for x or y.	M1	
	Obtain $6.69e^{\frac{1}{4}\pi i}$	A1	[5]

Q7.

(a)	Substitute $w = x + iy$ and state a correct equation in x and y	B1	
	Use $i^2 = -1$ and equate real parts	M1	
	Obtain $y = -2$	A1	
	Equate imaginary parts and solve for x	M1	
	Obtain $x = 2\sqrt{2}$, or equivalent, only	A1	[5]
(b)	Show a circle with centre 2i	B1	
	Show a circle with radius 2	B1	
	Show half line from -2 at $\frac{1}{4}\pi$ to real axis	B1	
	Shade the correct region	B1	
	Carry out a complete method for calculating the greatest value of $\|z\|$	M1	
	Obtain answer 3.70	A1	[6]

Q8.

(i)	Show that $a^2 + b^2 = (a + ib)(a - ib)$	B1	
	Show that $(a + ib - ki)^* = a - ib + ki$	B1	[2]
(ii)	Square both sides and express the given equation in terms of z and z^*	M1	
	Obtain a correct equation in any form, e.g. $(z - 10i)(z^* + 10i) = 4(z - 4i)(z^* + 4i)$	A1	
	Obtain the given equation	A1	
	Either express $\|z - 2i\| = 4$ in terms of z and z^* or reduce the given equation to the form $\|z - u\| = r$	M1	
	Obtain the given answer correctly	A1	[5]
(iii)	State that the locus is a circle with centre 2i and radius 5	B1	[1]

Q9.

(a)	*EITHER:* Solve for u or for v	M1	
	Obtain $u = \dfrac{2i-6}{1-2i}$ or $v = \dfrac{5}{1-2i}$, or equivalent	A1	
	Either: Multiply a numerator and denominator by conjugate of denominator, or equivalent *Or:* Set u or v equal to x + iy, obtain two equations by equating real and imaginary parts and solve for x or for y	M1	
	OR: Using $a + ib$ and $c + id$ for u and v, equate real and imaginary parts and obtain four equations in a, b, c and d	M1	
	Obtain $b + 2d = 2$, $a + 2c = 0$, $a + d = 0$ and $-b + c = 3$, or equivalent	A1	
	Solve for one unknown	M1	
	Obtain final answer $u = -2 - 2i$, or equivalent	A1	
	Obtain final answer $v = 1 + 2i$, or equivalent	A1	**[5]**
(b)	Show a circle with centre $-i$	B1	
	Show a circle with radius 1	B1	
	Show correct half line from 2 at an angle of $\frac{3}{4}\pi$ to the real axis	B1	
	Use a correct method for finding the least value of the modulus	M1	
	Obtain final answer $\dfrac{3}{\sqrt{2}} - 1$, or equivalent, e.g. 1.12 (allow 1.1)	A1	**[5]**

Q10.

(a)	Solve using formula, including simplification under square root sign	M1*	
	Obtain $\dfrac{-2 \pm 4i}{2(2-i)}$ or similarly simplified equivalents	A1	
	Multiply by $\dfrac{2+i}{2+i}$ or equivalent in at least one case	M1(d*M)	
	Obtain final answer $-\frac{4}{5} + \frac{3}{5}i$	A1	
	Obtain final answer $-i$	A1	[5]
(b)	Show w in first quadrant with modulus and argument relatively correct	B1	
	Show w^3 in second quadrant with modulus and argument relatively correct	B1	
	Show w^* in fourth quadrant with modulus and argument relatively correct	B1	
	Use correct method for area of triangle	M1	
	Obtain 10 by calculation	A1	[5]