Complex Numbers 2

Q1.

The complex number u is defined by $u = \dfrac{(1+2\mathrm{i})^2}{2+\mathrm{i}}$.

(i) Without using a calculator and showing your working, express u in the form $x + \mathrm{i}y$, where x and y are real. [4]

(ii) Sketch an Argand diagram showing the locus of the complex number z such that $|z - u| = |u|$. [3]

Q2.

Throughout this question the use of a calculator is not permitted.

The complex number u is defined by

$$u = \frac{1+2\mathrm{i}}{1-3\mathrm{i}}.$$

(i) Express u in the form $x + \mathrm{i}y$, where x and y are real. [3]

(ii) Show on a sketch of an Argand diagram the points A, B and C representing the complex numbers u, $1 + 2\mathrm{i}$ and $1 - 3\mathrm{i}$ respectively. [2]

(iii) By considering the arguments of $1 + 2\mathrm{i}$ and $1 - 3\mathrm{i}$, show that

$$\tan^{-1} 2 + \tan^{-1} 3 = \tfrac{3}{4}\pi.$$ [3]

Q3.

(a) The complex numbers u and w satisfy the equations

$$u - w = 4\mathrm{i} \quad \text{and} \quad uw = 5.$$

Solve the equations for u and w, giving all answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 2 + 2\mathrm{i}| \leqslant 2$, $\arg z \leqslant -\frac{1}{4}\pi$ and $\operatorname{Re} z \geqslant 1$, where $\operatorname{Re} z$ denotes the real part of z. [5]

(ii) Calculate the greatest possible value of $\operatorname{Re} z$ for points lying in the shaded region. [1]

Q4.

The complex number $1 + (\sqrt{2})\mathrm{i}$ is denoted by u. The polynomial $x^4 + x^2 + 2x + 6$ is denoted by $\mathrm{p}(x)$.

(i) Showing your working, verify that u is a root of the equation $\mathrm{p}(x) = 0$, and write down a second complex root of the equation. [4]

(ii) Find the other two roots of the equation $\mathrm{p}(x) = 0$. [6]

Q5.

(a) Without using a calculator, solve the equation $\mathrm{i}w^2 = (2 - 2\mathrm{i})^2$. [3]

(b) **(i)** Sketch an Argand diagram showing the region R consisting of points representing the complex numbers z where

$$|z - 4 - 4\mathrm{i}| \leqslant 2.$$ [2]

(ii) For the complex numbers represented by points in the region R, it is given that

$$p \leqslant |z| \leqslant q \quad \text{and} \quad \alpha \leqslant \arg z \leqslant \beta.$$

Find the values of p, q, α and β, giving your answers correct to 3 significant figures. [6]

Q6.

(a) Without using a calculator, solve the equation

$$3w + 2\mathrm{i}w^* = 17 + 8\mathrm{i},$$

where w^* denotes the complex conjugate of w. Give your answer in the form $a + b\mathrm{i}$. [4]

(b) In an Argand diagram, the loci

$$\arg(z - 2\mathrm{i}) = \tfrac{1}{6}\pi \quad \text{and} \quad |z - 3| = |z - 3\mathrm{i}|$$

intersect at the point P. Express the complex number represented by P in the form $re^{\mathrm{i}\theta}$, giving the exact value of θ and the value of r correct to 3 significant figures. [5]

Q7.

(a) The complex number w is such that $\text{Re}\, w > 0$ and $w + 3w^* = \text{i}w^2$, where w^* denotes the complex conjugate of w. Find w, giving your answer in the form $x + \text{i}y$, where x and y are real. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z - 2\text{i}| \leqslant 2$ and $0 \leqslant \arg(z + 2) \leqslant \frac{1}{4}\pi$. Calculate the greatest value of $|z|$ for points in this region, giving your answer correct to 2 decimal places. [6]

Q8.

The complex number z is defined by $z = a + \text{i}b$, where a and b are real. The complex conjugate of z is denoted by z^*.

(i) Show that $|z|^2 = zz^*$ and that $(z - k\text{i})^* = z^* + k\text{i}$, where k is real. [2]

In an Argand diagram a set of points representing complex numbers z is defined by the equation $|z - 10\text{i}| = 2|z - 4\text{i}|$.

(ii) Show, by squaring both sides, that

$$zz^* - 2\text{i}z^* + 2\text{i}z - 12 = 0.$$

Hence show that $|z - 2\text{i}| = 4$. [5]

(iii) Describe the set of points geometrically. [1]

Q9.

(a) The complex numbers u and v satisfy the equations

$$u + 2v = 2\text{i} \quad \text{and} \quad \text{i}u + v = 3.$$

Solve the equations for u and v, giving both answers in the form $x + \text{i}y$, where x and y are real. [5]

(b) On an Argand diagram, sketch the locus representing complex numbers z satisfying $|z + \text{i}| = 1$ and the locus representing complex numbers w satisfying $\arg(w - 2) = \frac{3}{4}\pi$. Find the least value of $|z - w|$ for points on these loci. [5]

Q10.

(a) Without using a calculator, use the formula for the solution of a quadratic equation to solve

$$(2 - \mathrm{i})z^2 + 2z + 2 + \mathrm{i} = 0.$$

Give your answers in the form $a + b\mathrm{i}$. [5]

(b) The complex number w is defined by $w = 2\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$. In an Argand diagram, the points A, B and C represent the complex numbers w, w^3 and w^* respectively (where w^* denotes the complex conjugate of w). Draw the Argand diagram showing the points A, B and C, and calculate the area of triangle ABC. [5]