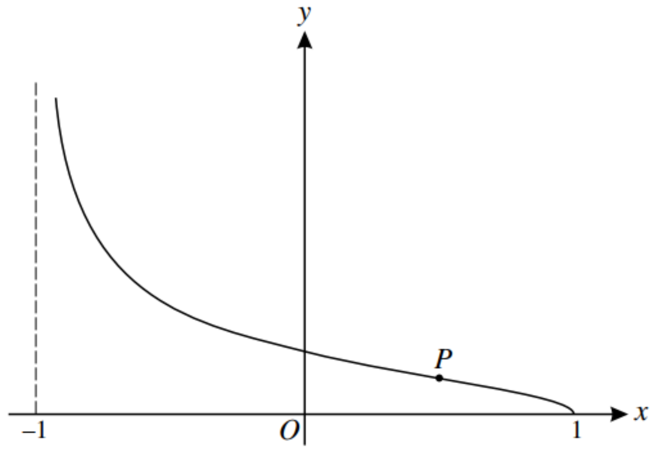
Differentiation 1

Q1.



The diagram shows the curve $y = \sqrt{\left(\frac{1-x}{1+x}\right)}$.

(i) By first differentiating $\frac{1-x}{1+x}$, obtain an expression for $\frac{dy}{dx}$ in terms of x. Hence show that the gradient of the normal to the curve at the point (x, y) is $(1+x)\sqrt{(1-x^2)}$. [5]

(ii) The gradient of the normal to the curve has its maximum value at the point P shown in the diagram. Find, by differentiation, the x-coordinate of P. [4]

Q2.

The equation of a curve is

$$x \ln y = 2x + 1.$$

(i) Show that $\frac{dy}{dx} = -\frac{y}{x^2}$. [4]

(ii) Find the equation of the tangent to the curve at the point where $y = 1$, giving your answer in the form $ax + by + c = 0$. [4]

Q3.

The parametric equations of a curve are

$$x = \frac{t}{2t+3}, \qquad y = e^{-2t}.$$

Find the gradient of the curve at the point for which $t = 0$. [5]

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Q4.

Find $\frac{dy}{dx}$ in each of the following cases:

(i) $y = \ln(1 + \sin 2x)$, [2]

(ii) $y = \frac{\tan x}{x}$. [2]

Q5.

The curve with equation

$$6e^{2x} + ke^{y} + e^{2y} = c,$$

where k and c are constants, passes through the point P with coordinates $(\ln 3, \ln 2)$.

(i) Show that $58 + 2k = c$. [2]

(ii) Given also that the gradient of the curve at P is -6, find the values of k and c. [5]

Q6.

The parametric equations of a curve are

$$x = \ln(\tan t), \quad y = \sin^2 t,$$

where $0 < t < \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of t. [4]

(ii) Find the equation of the tangent to the curve at the point where $x = 0$. [3]

Q7.

The curve $y = \frac{\ln x}{x^3}$ has one stationary point. Find the x-coordinate of this point. [4]

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Q8.

The parametric equations of a curve are

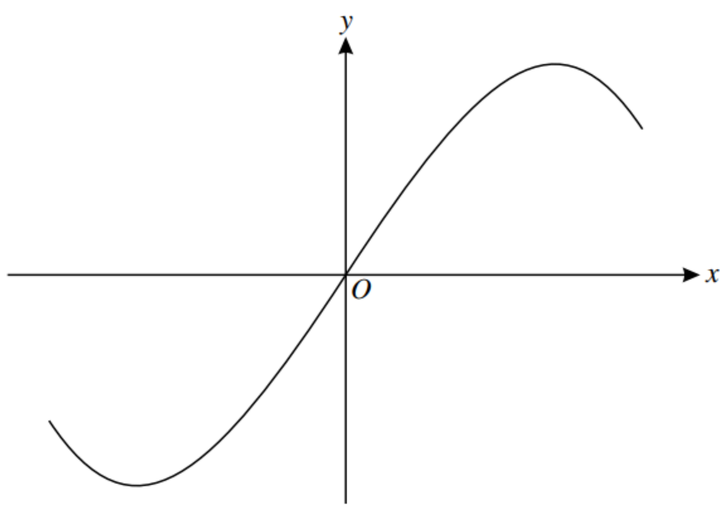
$$x = 3(1 + \sin^2 t), \quad y = 2\cos^3 t.$$

Find $\frac{dy}{dx}$ in terms of t, simplifying your answer as far as possible. [5]

Q9.

The equation of a curve is $y = \frac{e^{2x}}{1 + e^{2x}}$. Show that the gradient of the curve at the point for which $x = \ln 3$ is $\frac{9}{50}$. [4]

Q10.



The diagram shows the curve with parametric equations

$$x = \sin t + \cos t, \quad y = \sin^3 t + \cos^3 t,$$

for $\frac{1}{4}\pi < t < \frac{5}{4}\pi$.

(i) Show that $\frac{dy}{dx} = -3\sin t \cos t$. [3]

(ii) Find the gradient of the curve at the origin. [2]

(iii) Find the values of t for which the gradient of the curve is 1, giving your answers correct to 2 significant figures. [4]
