Integration 1 - Marking Scheme

Q1.

(i)	State correct expansion of $\cos(3x - x)$ or $\cos(3x + x)$	B1	
	Substitute expansions in $\frac{1}{2}(\cos 2x - \cos 4x)$, or equivalent	M1	
	Simplify and obtain the given identity correctly	A1	[3]
(ii)	Obtain integral $\frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x$	B1	
	Substitute limits correctly in an integral of the form $a\sin 2x + b\sin 4x$	M1	
	Obtain given answer following full, correct and exact working	A1	[3]

Q2.

(i)	State or imply the form $\frac{A}{x+1} + \frac{B}{x+3}$ and use a relevant method to find A or B	M1	
	Obtain $A = 1$, $B = -1$	A1	[2]
(ii)	Square the result of part **(i)** and substitute the fractions of part **(i)**	M1	
	Obtain the given answer correctly	A1	[2]
(iii)	Integrate and obtain $-\frac{1}{x+1} - \ln(x+1) + \ln(x+3) - \frac{1}{x+3}$	B3	
	Substitute limits correctly in an integral containing at least two terms of the correct form	M1	
	Obtain given answer following full and exact working	A1	[5]

Q3.

Integrate by parts and reach $\pm x^2 \cos x \pm \int 2x\cos x \, dx$	M1	
Obtain $-x^2 \cos x + \int 2x\cos x \, dx$, or equivalent	A1	
Complete the integration, obtaining $-x^2 \cos x + 2x\sin x + 2\cos x$, or equivalent	A1	
Substitute limits correctly, having integrated twice	M1	
Obtain the given answer correctly	A1	[5]

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Q4.

(i)	*EITHER*:	Divide by denominator and obtain quadratic remainder	M1	
		Obtain $A = 1$	A1	
		Use any relevant method to obtain B, C or D	M1	
		Obtain one correct answer	A1	
		Obtain $B = 2$, $C = 1$ and $D = -3$	A1	
	OR:	Reduce RHS to a single fraction and equate numerators, or equivalent	M1	
		Obtain $A = 1$	A1	
		Use any relevant method to obtain B, C or D	M1	
		Obtain one correct answer	A1	
		Obtain $B = 2$, $C = 1$ and $D = -3$	A1	[5]
		[SR: If $A = 1$ stated without working give B1.]		

(ii)	Integrate and obtain $x + 2\ln x - \frac{1}{x} - \frac{3}{2}\ln(2x-1)$, or equivalent	B3√	
	(The f.t. is on A, B, C, D. Give B2√ if only one error in integration; B1√ if two.)		
	Substitute limits correctly in the complete integral	M1	
	Obtain given answer correctly following full and exact working	A1	[5]

Q5.

(i)	State derivative $-e^{-x} - (-2)e^{-2x}$, or equivalent	B1 + B1	
	Equate derivative to zero and solve for x	M1	
	Obtain $p = \ln 2$, or exact equivalent	A1	[4]
(ii)	State indefinite integral $-e^{-x} - (-\frac{1}{2})e^{-2x}$, or equivalent	B1 + B1	
	Substitute limits $x = 0$ and $x = p$ correctly	M1	
	Obtain given answer following full and correct working	A1	[4]

Q6.

(i)	Use correct $\cos(A + B)$ formula to express $\cos 3\theta$ in terms of trig functions of 2θ and θ	M1	
	Use correct trig formulae and Pythagoras to express $\cos 3\theta$ in terms of $\cos\theta$	M1	
	Obtain a correct expression in terms of $\cos\theta$ in any form	A1	
	Obtain the given identity correctly	A1	[4]
	[SR: Give M1 for using correct formulae to express RHS in terms of $\cos\theta$ and $\cos 2\theta$, then M1A1 for expressing in terms of either only $\cos 3\theta$ and $\cos\theta$, or only $\cos 2\theta$, $\sin 2\theta$, $\cos\theta$, and $\sin\theta$, and A1 for obtaining the given identity correctly.]		
(ii)	Use identity and integrate, obtaining terms $\frac{1}{4}(\frac{1}{3}\sin 3\theta)$ and $\frac{1}{4}(3\sin\theta)$, or equivalent	B1 + B1	
	Use limits correctly in an integral of the form $k\sin 3\theta + l\sin\theta$	M1	
	Obtain answer $\frac{2}{3} - \frac{3}{8}\sqrt{3}$, or any exact equivalent	A1	[4]

Integration 1 - Marking Scheme

Q7.

(i)	State or imply $dx = 2\cos\theta\, d\theta$, or $\frac{dx}{d\theta} = 2\cos\theta$, or equivalent	B1	
	Substitute for x and dx throughout the integral	M1	
	Obtain the given answer correctly, having changed limits and shown sufficient working	A1	[3]
(ii)	Replace integrand by $2 - 2\cos 2\theta$, or equivalent	B1	
	Obtain integral $2\theta - \sin 2\theta$, or equivalent	B1√	
	Substitute limits correctly in an integral of the form $a\theta \pm b\sin 2\theta$, where ab Þ 0	M1	
	Obtain answer $\frac{1}{3}\pi - \frac{\sqrt{3}}{2}$ or exact equivalent	A1	[4]
	[The f.t. is on integrands of the form $a + c\cos 2\theta$, where ac Þ 0.]		

Q8.

(i)	Use correct product rule	M1	
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and find non-zero x	M1	
	Obtain $x = \exp(-\frac{1}{3})$, or equivalent	A1	
	Obtain $y = -1/(3e)$, or any ln-free equivalent	A1	[5]
(ii)	Integrate and reach $kx^4 \ln x + l\int x^4 . \frac{1}{x}\,dx$	M1	
	Obtain $\frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3\,dx$	A1	
	Obtain integral $\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$, or equivalent	A1	
	Use limits $x = 1$ and $x = 2$ correctly, having integrated twice	M1	
	Obtain answer $4\ln 2 - \frac{15}{16}$, or exact equivalent	A1	[5]

Q9.

(i)	Obtain derivative of form $k\cos 3x \sin 3x$, any constant k	M1	
	Obtain $-24\cos 3x \sin 3x$ or unsimplified equivalent	A1	
	Obtain $-6\sqrt{3}$ or exact equivalent	A1	[3]
(ii)	Express integrand in the form $a + b\cos 6x$, where $ab \neq 0$	M1	
	Obtain $2 + 2\cos 6x$ o.e.	A1	
	Obtain $2x + \frac{1}{3}\sin 6x$ or equivalent, condoning absence of $+ c$, ft on a, b	A1√	[3]

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Q10.

State or imply form $\frac{A}{2x+1}+\frac{B}{x+2}$	B1	
Use relevant method to find A or B	M1	
Obtain $\frac{4}{2x+1}-\frac{1}{x+2}$	A1	
Integrate and obtain $2\ln(2x+1)-\ln(x+2)$ (ft on their A, B)	B1√B1√	
Apply limits to integral containing terms $a\ln(2x+1)$ and $b\ln(x+2)$ and apply a law of logarithms correctly.	M1	
Obtain given answer $\ln 50$ correctly	A1	[7]