

Integration 1



Q1.

- (i) Using the expansions of $\cos(3x - x)$ and $\cos(3x + x)$, prove that

$$\frac{1}{2}(\cos 2x - \cos 4x) \equiv \sin 3x \sin x. \quad [3]$$

- (ii) Hence show that

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin 3x \sin x \, dx = \frac{1}{8}\sqrt{3}. \quad [3]$$

Q2.

- (i) Express $\frac{2}{(x+1)(x+3)}$ in partial fractions. [2]

- (ii) Using your answer to part (i), show that

$$\left(\frac{2}{(x+1)(x+3)}\right)^2 \equiv \frac{1}{(x+1)^2} - \frac{1}{x+1} + \frac{1}{x+3} + \frac{1}{(x+3)^2}. \quad [2]$$

- (iii) Hence show that $\int_0^1 \frac{4}{(x+1)^2(x+3)^2} \, dx = \frac{7}{12} - \ln \frac{3}{2}$. [5]
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Q3.

Show that $\int_0^\pi x^2 \sin x \, dx = \pi^2 - 4$. [5]

Q4.

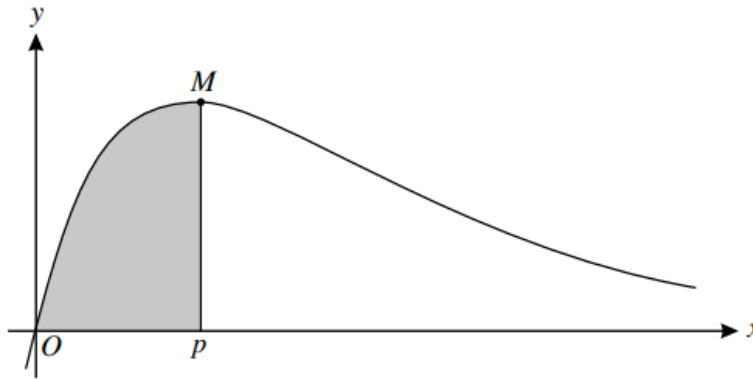
- (i) Find the values of the constants A , B , C and D such that

$$\frac{2x^3 - 1}{x^2(2x - 1)} \equiv A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{2x - 1}. \quad [5]$$

- (ii) Hence show that

$$\int_1^2 \frac{2x^3 - 1}{x^2(2x - 1)} \, dx = \frac{3}{2} + \frac{1}{2} \ln\left(\frac{16}{27}\right). \quad [5]$$

Q5.



The diagram shows the curve $y = e^{-x} - e^{-2x}$ and its maximum point M . The x -coordinate of M is denoted by p .

- (i) Find the exact value of p . [4]
- (ii) Show that the area of the shaded region bounded by the curve, the x -axis and the line $x = p$ is equal to $\frac{1}{8}$. [4]
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Q6.

- (i) Prove the identity $\cos 3\theta \equiv 4 \cos^3 \theta - 3 \cos \theta$. [4]
- (ii) Using this result, find the exact value of

$$\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \cos^3 \theta \, d\theta. \quad [4]$$

Q7.

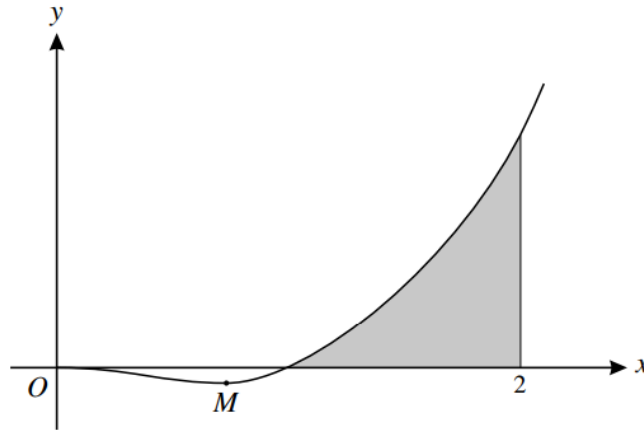
Let $I = \int_0^1 \frac{x^2}{\sqrt{4-x^2}} \, dx$.

- (i) Using the substitution $x = 2 \sin \theta$, show that

$$I = \int_0^{\frac{1}{2}\pi} 4 \sin^2 \theta \, d\theta. \quad [3]$$

- (ii) Hence find the exact value of I . [4]
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Q8.



The diagram shows the curve $y = x^3 \ln x$ and its minimum point M .

(i) Find the exact coordinates of M . [5]

(ii) Find the exact area of the shaded region bounded by the curve, the x -axis and the line $x = 2$. [5]

Q9.

It is given that $f(x) = 4 \cos^2 3x$.

(i) Find the exact value of $f'(\frac{1}{9}\pi)$. [3]

(ii) Find $\int f(x) dx$. [3]

Q10.

Show that $\int_0^7 \frac{2x+7}{(2x+1)(x+2)} dx = \ln 50$. [7]
