Integration 2

Q1.

The integral I is defined by $I = \int_0^2 4t^3 \ln(t^2 + 1)\,dt$.

(i) Use the substitution $x = t^2 + 1$ to show that $I = \int_1^5 (2x - 2)\ln x\,dx$. [3]

(ii) Hence find the exact value of I. [5]

Q2.

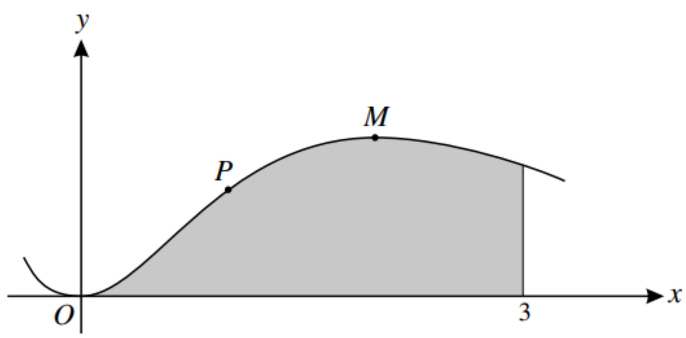
(i) Prove the identity $\cos 4\theta + 4\cos 2\theta \equiv 8\cos^4\theta - 3$. [4]

(ii) Hence

(a) solve the equation $\cos 4\theta + 4\cos 2\theta = 1$ for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$, [3]

(b) find the exact value of $\int_0^{\frac{1}{4}\pi} \cos^4\theta\,d\theta$. [3]

Q3.



The diagram shows the curve $y = x^2e^{-x}$.

(i) Show that the area of the shaded region bounded by the curve, the x-axis and the line $x = 3$ is equal to $2 - \frac{17}{e^3}$. [5]

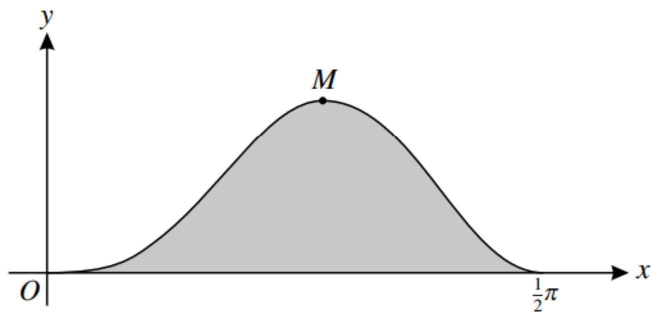
(ii) Find the x-coordinate of the maximum point M on the curve. [4]

(iii) Find the x-coordinate of the point P at which the tangent to the curve passes through the origin. [2]

Q4.

Show that $\int_0^1 (1-x)e^{-\frac{1}{2}x}\,dx = 4e^{-\frac{1}{2}} - 2.$ [5]

Q5.

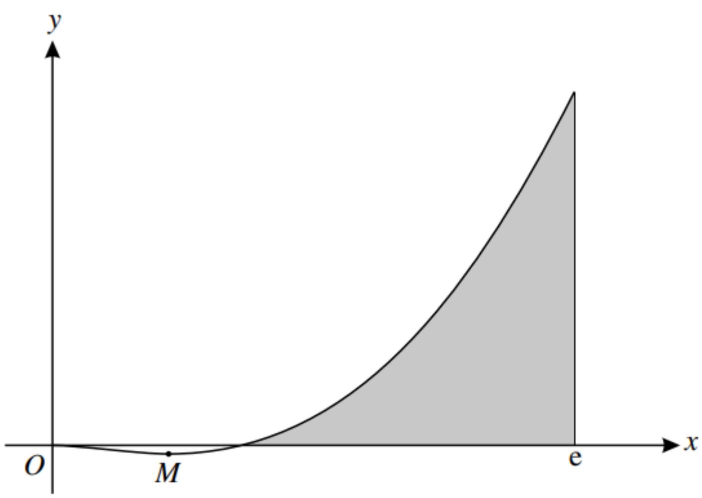


The diagram shows the curve $y = 5\sin^3 x\cos^2 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Find the x-coordinate of M. [5]

(ii) Using the substitution $u = \cos x$, find by integration the area of the shaded region bounded by the curve and the x-axis. [5]

Q6.



The diagram shows the curve $y = x^2 \ln x$ and its minimum point M.

(i) Find the exact values of the coordinates of M. [5]

(ii) Find the exact value of the area of the shaded region bounded by the curve, the x-axis and the line $x = e$. [5]

Q7.

(i) Use the substitution $u = \tan x$ to show that, for $n \neq -1$,

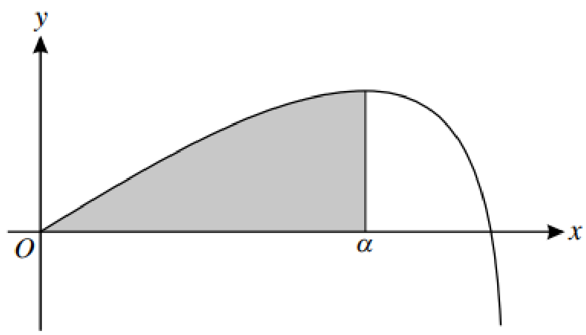
$$\int_0^{\frac{1}{4}\pi} (\tan^{n+2} x + \tan^n x)\, dx = \frac{1}{n+1}.$$ [4]

(ii) Hence find the exact value of

(a) $\int_0^{\frac{1}{4}\pi} (\sec^4 x - \sec^2 x)\, dx$, [3]

(b) $\int_0^{\frac{1}{4}\pi} (\tan^9 x + 5\tan^7 x + 5\tan^5 x + \tan^3 x)\, dx$. [3]

Q8.



The diagram shows the curve

$$y = 8\sin\tfrac{1}{2}x - \tan\tfrac{1}{2}x$$

for $0 \leqslant x < \pi$. The x-coordinate of the maximum point is α and the shaded region is enclosed by the curve and the lines $x = \alpha$ and $y = 0$.

(i) Show that $\alpha = \frac{2}{3}\pi$. [3]

(ii) Find the exact value of the area of the shaded region. [4]

Q9.

By first expressing $\dfrac{4x^2 + 5x + 3}{2x^2 + 5x + 2}$ in partial fractions, show that

$$\int_0^4 \frac{4x^2 + 5x + 3}{2x^2 + 5x + 2}\, dx = 8 - \ln 9.$$ [10]

Q10.

Let $I = \int_2^5 \frac{5}{x + \sqrt{(6-x)}} \, dx$.

(i) Using the substitution $u = \sqrt{(6-x)}$, show that

$$I = \int_1^2 \frac{10u}{(3-u)(2+u)} \, du.$$ [4]

(ii) Hence show that $I = 2\ln\left(\frac{9}{2}\right)$. [6]