Integration 2 - Marking Scheme

Q1.

(i)	State or imply $dx = 2t\,dt$ or equivalent	B1	
	Express the integral in terms of x and dx	M1	
	Obtain given answer $\int_1^5 (2x-2)\ln x\,dx$, including change of limits **AG**	A1	[3]
(ii)	Attempt integration by parts obtaining $(ax^2+bx)\ln x \pm \int (ax^2+bx)\frac{1}{x}dx$ or equivalent	M1	
	Obtain $(x^2-2x)\ln x - \int (x^2-2x)\frac{1}{x}dx$ or equivalent	A1	
	Obtain $(x^2-2x)\ln x - \frac{1}{2}x^2 + 2x$	A1	
	Use limits correctly having integrated twice	M1	
	Obtain $15 \ln 5 - 4$ or exact equivalent	A1	[5]
	[Equivalent for M1 is $(2x-2)(ax\ln x + bx) - \int (ax\ln x + bx)\,2dx$]		

Q2.

(i)		Express $\cos 4\theta$ as $2\cos^2 2\theta - 1$ or $\cos^2 2\theta - \sin^2 2\theta$ or $1 - 2\sin^2 2\theta$	B1	
		Express $\cos 4\theta$ in terms of $\cos\theta$	M1	
		Obtain $8\cos^4\theta - 8\cos^2\theta + 1$	A1	
		Use $\cos 2\theta = 2\cos^2\theta - 1$ to obtain given answer $8\cos^4\theta - 3$ **AG**	A1	[4]
(ii)	**(a)**	State or imply $\cos^4\theta = \frac{1}{2}$	B1	
		Obtain 0.572	B1	
		Obtain –0.572	B1	[3]
	(b)	Integrate and obtain form $k_1\theta + k_2 \sin 4\theta + k_3 \sin 2\theta$	M1	
		Obtain $\frac{3}{8}\theta + \frac{1}{32}\sin 4\theta + \frac{1}{4}\sin 2\theta$	A1	
		Obtain $\frac{3}{32}\pi + \frac{1}{4}$ following completely correct work	A1	[3]

Integration 2 - Marking Scheme

Q3.

(i)	Attempt integration by parts and reach $\pm x^2e^{-x} \pm \int 2xe^{-x}dx$	M1*	
	Obtain $-x^2e^{-x} + \int 2xe^{-x}dx$, or equivalent	A1	
	Integrate and obtain $-x^2e^{-x} - 2xe^{-x} - 2e^{-x}$, or equivalent	A1	
	Use limits $x = 0$ and $x = 3$, having integrated by parts twice	M1(dep*)	
	Obtain the given answer correctly	A1	[5]
(ii)	Use correct product or quotient rule	M1	
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and solve for non-zero x	M1	
	Obtain $x = 2$ with no errors send	A1	[4]
(iii)	Carry out a complete method for finding the x-coordinate of P	M1	
	Obtain answer $x = 1$	A1	[2]

Q4.

Attempt integration by parts and reach $k(1-x)e^{-\frac{1}{2}x} \pm k\int e^{-\frac{1}{2}x}dx$, or equivalent	M1	
Obtain $-2(1-x)e^{-\frac{1}{2}x} - 2\int e^{-\frac{1}{2}x}dx$, or equivalent	A1	
Integrate and obtain $-2(1-x)e^{-\frac{1}{2}x} + 4e^{-\frac{1}{2}x}$, or equivalent	A1	
Use limits $x = 0$ and $x = 1$, having integrated twice	M1	
Obtain the given answer correctly	A1	[5]

Q5.

8	**(i)**	Use product and chain rule	M1	
		Obtain correct derivative in any form, e.g. $15\sin^2 x\cos^3 x - 10\sin^4 x\cos x$	A1	
		Equate derivative to zero and obtain a relevant equation in one trigonometric function	M1	
		Obtain $2\tan^2 x = 3$, $5\cos^2 x = 2$, or $5\sin^2 x = 3$	A1	
		Obtain answer $x = 0.886$ radians	A1	[5]
	(ii)	State or imply $du = -\sin x\, dx$, or $\frac{du}{dx} = -\sin x$, or equivalent	B1	
		Express integral in terms of u and du	M1	
		Obtain $\pm\int 5(u^2 - u^4)du$, or equivalent	A1	
		Integrate and use limits $u = 1$ and $u = 0$ (or $x = 0$ and $x = \frac{1}{2}\pi$)	M1	
		Obtain answer $\frac{2}{3}$, or equivalent, with no errors seen	A1	[5]

Integration 2 - Marking Scheme

Q6.

(i)	Use product rule	M1	
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = e^{-\frac{1}{2}}$, or equivalent	A1	
	Obtain answer $y = -\frac{1}{2}e^{-1}$, or equivalent	A1	[5]
(ii)	Attempt integration by parts reaching $kx^3 \ln x \pm k \int x^3 . \frac{1}{x} dx$	M1*	
	Obtain $\frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 \, dx$, or equivalent	A1	
	Integrate again and obtain $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$, or equivalent	A1	
	Use limits $x = 1$ and $x = e$, having integrated twice	M1(dep*)	
	Obtain answer $\frac{1}{9}(2e^3 + 1)$, or exact equivalent	A1	[5]

[SR: An attempt reaching $ax^2 (x \ln x - x) + b \int 2x(x \ln x - x) \, dx$ scores M1. Then give the first A1 for $I = x^2 (x \ln x - x) - 2I + \int 2x^2 \, dx$, or equivalent.]

Integration 2 - Marking Scheme

Q7.

10	**(i)** State or imply $\frac{du}{dx} = \sec^2 x$	B1	
	Express integrand in terms of u and du	M1	
	Integrate to obtain $\frac{u^{n+1}}{n+1}$ or equivalent	A1	
	Substitute correct limits correctly to confirm given result $\frac{1}{n+1}$	A1	[4]
	(ii) **(a)** Use $\sec^2 x = 1 + \tan^2 x$ twice	M1	
	Obtain integrand $\tan^4 x + \tan^2 x$	A1	
	Apply result from part **(i)** to obtain $\frac{1}{3}$	A1	[3]
	Or Use $\sec^2 x = 1 + \tan^2 x$ and the substitution from **(i)**	M1	
	Obtain $\int u^2 \mathrm{d}u$	A1	
	Apply limits correctly and obtain $\frac{1}{3}$	A1	
	(b) Arrange, perhaps implied, integrand to $t^9 + t^7 + 4(t^7 + t^5) + t^5 + t^3$	B1	
	Attempt application of result from part **(i)** at least twice	M1	
	Obtain $\frac{1}{8} + \frac{4}{6} + \frac{1}{4}$ and hence $\frac{25}{24}$ or exact equivalent	A1	[3]

Integration 2 - Marking Scheme

Q8.

5	**(i)**	Differentiate to obtain $4\cos\frac{1}{2}x - \frac{1}{2}\sec^2\frac{1}{2}x$	B1	
		Equate to zero and find value of $\cos\frac{1}{2}x$	M1	
		Obtain $\cos\frac{1}{2}x = \frac{1}{2}$ and confirm $\alpha = \frac{2}{3}\pi$	A1	[3]
	(ii)	Integrate to obtain $-16\cos\frac{1}{2}x \ldots$	B1	
		$\ldots\ +2\ln\cos\frac{1}{2}x$ or equivalent	B1	
		Using limits 0 and $\frac{2}{3}\pi$ in $a\cos\frac{1}{2}x + b\ln\cos\frac{1}{2}x$	M1	
		Obtain $8 + 2\ln\frac{1}{2}$ or exact equivalent	A1	[4]

Q9.

State or imply form $A + \frac{B}{2x+1} + \frac{C}{x+2}$	B1	
State or obtain $A = 2$	B1	
Use correct method for finding B or C	M1	
Obtain $B = 1$	A1	
Obtain $C = -3$	A1	
Obtain $2x + \frac{1}{2}\ln(2x+1) - 3\ln(x+2)$ [Deduct B1√ for each error or omission]	B3√	
Substitute limits in expression containing $a\ln(2x + 1) + b\ln(x + 2)$	M1	
Show full and exact working to confirm that $8 + \frac{1}{2}\ln 9 - 3\ln 6 + 3\ln 2$, or an equivalent expression, simplifies to given result $8 - \ln 9$	A1	[10]

[SR: If A omitted from the form of fractions, give B0B0M1A0A0 in (**i**); B0√B1√B1√M1A0 in (**ii**).]

[SR: For a solution starting with $\frac{M}{2x+1} + \frac{Nx}{x+2}$ or $\frac{Px}{2x+1} + \frac{Q}{x+2}$, give B0B0M1A0A0 in (**i**); B1√B1√B1√, if recover correct form, M1A0 in (**ii**).]

[SR: For a solution starting with $\frac{B}{2x+1} + \frac{Dx+E}{x+2}$, give M1A1 for one of $B = 1$, $D = 2$, $E = 1$ and A1 for the other two constants; then give B1B1 for $A = 2$, $C = -3$.]

[SR: For a solution starting with $\frac{Fx+G}{2x+1} + \frac{C}{x+2}$, give M1A1 for one of $C = -3$, $F = 4$, $G = 3$ and A1 for the other constants or constant; then give B1B1 for $A = 2$, $B = 1$.]

Integration 2 - Marking Scheme

Q10.

(i)	State or imply $2u\,du = -dx$, or equivalent	B1	
	Substitute for x and dx throughout	M1	
	Obtain integrand $\dfrac{-10u}{6-u^2+u}$, or equivalent	A1	
	Show correct working to justify the change in limits and obtain the given answer correctly	A1	[4]
(ii)	State or imply the form of fractions $\dfrac{A}{3-u}+\dfrac{B}{2+u}$ and use a relevant method to find A or B	M1	
	Obtain $A = 6$ and $B = -4$	A1	
	Integrate and obtain $-6\ln(3-u)-4\ln(2+u)$, or equivalent	A1√ + A1√	
	Substitute limits correctly in an integral of the form $a\ln(3-u)+b\ln(2+u)$	M1	
	Obtain the given answer correctly having shown sufficient working	A1	[6]
	[The f.t. is on A and B.]		