

# Trigonometry 1



Q1.

Solve the equation

$$\sin \theta = 2 \cos 2\theta + 1,$$

giving all solutions in the interval  $0^\circ \leq \theta \leq 360^\circ$ . [6]

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Q2.

It is given that  $\cos a = \frac{3}{5}$ , where  $0^\circ < a < 90^\circ$ . Showing your working and without using a calculator to evaluate  $a$ ,

(i) find the exact value of  $\sin(a - 30^\circ)$ , [3]

(ii) find the exact value of  $\tan 2a$ , and hence find the exact value of  $\tan 3a$ . [4]

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Q3.

Solve the equation

$$\tan(45^\circ - x) = 2 \tan x,$$

giving all solutions in the interval  $0^\circ < x < 180^\circ$ . [5]

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Q4.

Solve the equation

$$\cos(\theta + 60^\circ) = 2 \sin \theta,$$

giving all solutions in the interval  $0^\circ \leq \theta \leq 360^\circ$ . [5]

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Q5.

(i) Express  $(\sqrt{6}) \cos \theta + (\sqrt{10}) \sin \theta$  in the form  $R \cos(\theta - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . Give the value of  $\alpha$  correct to 2 decimal places. [3]

(ii) Hence, in each of the following cases, find the smallest positive angle  $\theta$  which satisfies the equation

(a)  $(\sqrt{6}) \cos \theta + (\sqrt{10}) \sin \theta = -4$ , [2]

(b)  $(\sqrt{6}) \cos \frac{1}{2}\theta + (\sqrt{10}) \sin \frac{1}{2}\theta = 3$ . [4]

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Q6.

(i) Prove the identity  $\cos 4\theta + 4 \cos 2\theta \equiv 8 \cos^4 \theta - 3$ . [4]

(ii) Hence

(a) solve the equation  $\cos 4\theta + 4 \cos 2\theta = 1$  for  $-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi$ , [3]

(b) find the exact value of  $\int_0^{\frac{1}{4}\pi} \cos^4 \theta \, d\theta$ . [3]

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Q7.

Solve the equation

$$\cos \theta + 4 \cos 2\theta = 3,$$

giving all solutions in the interval  $0^\circ \leq \theta \leq 180^\circ$ . [5]

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Q8.

(i) Show that the equation

$$\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = k$$

can be written in the form

$$(2\sqrt{3})(1 + \tan^2 \theta) = k(1 - 3 \tan^2 \theta). [4]$$

(ii) Hence solve the equation

$$\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = 3\sqrt{3},$$

giving all solutions in the interval  $0^\circ \leq \theta \leq 180^\circ$ . [3]

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Q9.

(i) Express  $\cos x + 3 \sin x$  in the form  $R \cos(x - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ , giving the exact value of  $R$  and the value of  $\alpha$  correct to 2 decimal places. [3]

(ii) Hence solve the equation  $\cos 2\theta + 3 \sin 2\theta = 2$ , for  $0^\circ < \theta < 90^\circ$ . [5]

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Q10.

(i) Express  $8 \cos \theta + 15 \sin \theta$  in the form  $R \cos(\theta - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . Give the value of  $\alpha$  correct to 2 decimal places. [3]

(ii) Hence solve the equation  $8 \cos \theta + 15 \sin \theta = 12$ , giving all solutions in the interval  $0^\circ < \theta < 360^\circ$ . [4]

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