Vectors 1 - Marking Scheme

Q1.

11(a)	Express general point of at least one line correctly in component form, i.e. $(1 + a\lambda, 2 + 2\lambda, 1 - \lambda)$ or $(2 + 2\mu, 1 - \mu, -1 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	May be implied $1 + a\lambda = 2 + 2\mu$ $2 + 2\lambda = 1 - \mu$ $1 - \lambda = -1 + \mu$
	Obtain $\lambda = -3$ or $\mu = 5$	**A1**	
	Obtain $a = -\frac{11}{3}$	**A1**	Allow $a = -3.667$
	State that the point of intersection has position vector $12\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$	**A1**	Allow coordinate form $(12, -4, 4)$
		5	

11(b)	Use correct process for finding the scalar product of direction vectors for the two lines	**M1**	$(a, 2, -1) . (2, -1, 1) = 2a - 2 - 1$ or $2a - 3$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\pm\frac{1}{6}$	***M1**	
	State a correct equation in a in any form, e.g. $\frac{2a - 2 - 1}{\sqrt{6}\sqrt{(a^2 + 5)}} = \pm\frac{1}{6}$	**A1**	
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a - 3)^2$ to produce 3 terms e.g. $36(2a - 3)^2 = 6(a^2 + 5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a - 49)(a - 1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$

11(b)	**Alternative method for question 11(b)**		
	$\cos(\theta) = [\lvert a^2 + 2^2 + (-1)^2\rvert^2 + \lvert 2^2 + (-1)^2 + 1^2\rvert^2 - \lvert(a - 2)^2 + 3^2 + (-2)^2\rvert^2] / [2\lvert a^2 + 2^2 + (-1)^2\rvert . \lvert 2^2 + (-1)^2 + 1^2\rvert]$	**M1**	Use of cosine rule. Must be correct vectors.
	Equate the result to $\pm\frac{1}{6}$	***M1** **A1**	Allow M1* here for any two vectors
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a - 3)^2$ to produce 3 terms e.g. $36(2a - 3)^2 = 6(a^2 + 5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a - 49)(a - 1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$
		6	

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Q2.

8(a)	Obtain $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$	**B1**	Or equivalent seen or implied
	Use the correct process for calculating the modulus of both vectors to obtain AB and CD	**M1**	$AB = \sqrt{24}, CD = \sqrt{6}$
	Using exact values, verify that $AB = 2CD$	**A1**	Obtain given statement from correct work Allow from $BA = 2DC$, OE
		3	
8(b)	Use the correct process to calculate the scalar product of the relevant vectors (*their* $\overrightarrow{AB}$ and $\overrightarrow{CD}$)	**M1**	$\begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$
	Divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer $99.6°$ (or 1.74 radians) or better	**A1**	Do not ISW if go on to subtract from 180° (99.594…, 1.738...) Accept 260.4°
		3	

Q3.

7(a)	Express general point of a line in component form, e.g. $(1 + 2s, 3 - s, 2 + 3s)$ or $(2 + t, 1 - t, 4 + 4t)$	**B1**	
	Equate at least two pairs of components and solve for s or for t	**M1**	
	Obtain correct answer for s or for t (possible answers are -1, 6, $\frac{2}{5}$ for s and -3, 4, $-\frac{1}{5}$ for t)	**A1**	
	Verify that all three component equations are not satisfied	**A1**	
	Show that the lines are not parallel and are thus skew	**A1**	
		5	
7(b)	Carry out correct process for evaluating the scalar product of the direction vectors	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer 19.1° or 0.333 radians	**A1**	
		3	

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Q4.

8(a)	State or imply $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$	B1	OE. Allow $\pm$
	Use the correct process to calculate the scalar product of a pair of relevant vectors, e.g. their $\overrightarrow{AB}$ and a direction vector for l	M1	$(2+2-3=1)$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli of the two vectors and evaluate the inverse cosine of the result	M1	$\cos^{-1}\left(\frac{1}{\sqrt{6}\sqrt{14}}\right)$
	Obtain answer $83.7°$ or 1.46 radians	A1	Or answers rounding to $83.7°$ or 1.46 radians
		4	

8(b)	State or imply $\pm\ \overrightarrow{AP}$ and $\pm\ \overrightarrow{BP}$ in component form, i.e. $(1+\lambda, 1-2\lambda, \lambda)$ and $(-1+\lambda, 2-2\lambda, 3+\lambda)$, or equivalent	B1	
	Form an equation in λ by equating moduli or by using $\cos BAP = \cos ABP$	*M1	
	Obtain a correct equation in any form $(1+\lambda)^2+(1-2\lambda)^2+\lambda^2=(\lambda-1)^2+(2-2\lambda)^2+(\lambda+3)^2$	A1	Or $(1+\lambda)\sqrt{14-4\lambda+6\lambda^2}=(13-\lambda)\sqrt{2-2\lambda+6\lambda^2}$ $(83\lambda^3-528\lambda^2+207\lambda-162=0)$
	Solve for λ and obtain position vector	DM1	$[\lambda = 6]$
	Obtain correct position vector for P in any form, e.g. $(8, -9, 7)$ or $8\mathbf{i}-9\mathbf{j}+7\mathbf{k}$	A1	Accept coordinates
		5	

Q5.

11(a)	Show that $OA = OB = \sqrt{5}$	B1	CWO
	Evaluate the scalar product of the correct position vectors	M1	e.g. $(0-1+0)$ Condone of using AO and/or BO
	Divide *their* scalar product by the product of the moduli of *their* vectors and evaluate the inverse cosine of the result	M1	Much reach an angle. The question asks for the use of scalar product, so alternative methods (e.g. cosine rule) are not accepted.
	Obtain answer $101.5°$	A1	The question asks for an answer in degrees. Accept $102°$ or better. Mark radians (1.77) as a misread. Do not ISW: $78.5°$ as final answer scores A0.
		4	

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11(b)	State or imply M has position vector $\mathbf{i} - \mathbf{k}$	**B1**	OE
	Taking a general point of OM to have position vector $\lambda\mathbf{i} - \lambda\mathbf{k}$, express $AP = \sqrt{7}\ OA$ as an equation in λ	***M1**	$\lambda(\textit{their } \overline{OM})$
	State a correct equation in any form	**A1**	e.g. $\sqrt{(-2+\lambda)^2 + 1 + (-\lambda)^2} = \sqrt{7}\sqrt{5}$
	Reduce to $\lambda^2 - 2\lambda - 15 = 0$	**A1**	OE
	Solve a quadratic and state a position vector	**DM1**	
	Obtain answers $5\mathbf{i} - 5\mathbf{k}$ and $-3\mathbf{i} + 3\mathbf{k}$	**A1**	Accept coordinates
	Alternative method for Question 11(b)		
	State or imply that $OP = \gamma\sqrt{2}$	**B1**	
	State or imply that $\cos\frac{1}{2}AOB = \sqrt{\frac{2}{5}}$ and use cosine rule to form an equation in γ	***M1**	Allow $\cos\frac{1}{2}AOB = 0.632...$
	State a correct equation in any form	**A1**	e.g. $35 = 5 + 2\gamma^2 - 2\sqrt{5}.\gamma\sqrt{2}.\frac{\sqrt{2}}{\sqrt{5}}$
	Reduce to $\gamma^2 - 2\gamma - 15 = 0$	**A1**	OE
	Solve a quadratic and state a position vector	**DM1**	
	Obtain answers $5\mathbf{i} - 5\mathbf{k}$ and $-3\mathbf{i} + 3\mathbf{k}$	**A1**	Accept coordinates

11(b)	**Alternative method for Question 11(b)**		
	State or imply M has position vector $\mathbf{i} - \mathbf{k}$	**B1**	OE
	State or imply that $AM = \sqrt{3}$	**B1**	
	Use Pythagoras to find MP	***M1**	$MP = \sqrt{35 - (AM)^2}$
	Obtain $MP = 4\sqrt{2}$	**A1**	
	Correct method to find a position vector	**DM1**	$(\mathbf{i} - \mathbf{k}) \pm 4(\mathbf{i} - \mathbf{k})$
	Obtain answers $5\mathbf{i} - 5\mathbf{k}$ and $-3\mathbf{i} + 3\mathbf{k}$	**A1**	Accept coordinates
		6	

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Q6.

9(a)	State or imply $\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$	**B1**	OE
	Carry out a correct method to find $\overrightarrow{OD}$	**M1**	
	Obtain answer $-4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$	**A1**	OE
		3	
9(b)	State $\mathbf{r} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	**B1FT**	OE. The FT is on $\overrightarrow{AB}$.
		1	
9(c)	For a general point *P* on *AB*, state $\overrightarrow{CP}$ or $\overrightarrow{DP}$ in component form, e.g. $\overrightarrow{CP} = (3 - 2\lambda, -\lambda, -6 + 2\lambda)$	***M1**	
	Equate a relevant scalar product to zero *or* equate derivative of $\lvert\overrightarrow{CP}\rvert$ to zero *or* use Pythagoras in a relevant triangle and solve for λ	**DM1**	
	Obtain $\lambda = 2$	**A1**	
	Show the perpendicular is of length 3	**A1**	
	Carry out a correct method to find the area of *ABCD* and obtain the answer 18	**A1**	
	Alternative method for Question 9(c)		
	Use a scalar product to find the projection *CN* (or *DN*) of *BC* (or *AD*) on *CD*	***M1**	
	Obtain $CN = 3$ (or $DN = 3$)	**A1**	
	Use Pythagoras to obtain *BN* (or *AN*)	**DM1**	
9(c) cont'd	Obtain answer 3	**A1**	
	Carry out a correct method to find the area of *ABCD* and obtain the answer 18	**A1**	
		5	

Q7.

9(a)	Use correct method to evaluate the scalar product of relevant vectors	**M1**	$(-4 - 2 + 6)$
	Obtain answer zero and deduce the given statement	**A1**	Need a conclusion or a statement in advance that the scalar product will be zero.
		2	
9(b)	Express general point of *l* or *m* in component form, e.g. $(3 + 4s, 2 - s, 5 + 3s)$ or $(1 - t, -1 + 2t, -2 + 2t)$	**B1**	
	Equate at least two pairs of components and solve for *s* or for *t*	**M1**	
	Obtain correct answer $s = -1$ and $t = 2$	**A1**	
	Verify that all three equations are satisfied	**A1**	
	State position vector of the intersection $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$, or equivalent	**A1**	Can come from 1 correct value and no contradictory statement.
		5	

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9(c)	Taking a general point P on m, form an equation in t by ***either*** equating a relevant scalar product to zero, ***or*** equating the derivative of $\lvert\overline{OP}\rvert$ to zero, ***or*** taking a specific point Q on m, e.g. (1, – 1, – 2), using Pythagoras in triangle OPQ	***M1**	e.g. $\begin{pmatrix}1-t\\-1+2t\\-2+2t\end{pmatrix}.\begin{pmatrix}-1\\2\\2\end{pmatrix}=0$
	Obtain $t=\frac{7}{9}$	**A1**	
	Carry out correct method to find OP	**DM1**	
	Obtain $\frac{\sqrt{5}}{3}$	**A1**	Obtain the **given answer** from full and correct working.
	Alternative method for question 9(c)		
	Take a specific point Q on m, e.g. (– 1, 3, 2) and use a scalar product to find QN, the projection of OQ on m	***M1**	
	Obtain $QN=\frac{11}{3}$, or equivalent	**A1**	
	Use Pythagoras to obtain ON	**DM1**	
	Obtain the given answer correctly	**A1**	
		4	

Q8.

10(a)	Obtain direction vector $-\mathbf{i}+\mathbf{j}+2\mathbf{k}$, or equivalent	**B1**	Accept answers as column vectors throughout.
	Use a correct method to form a vector equation	**M1**	
	State answer $\mathbf{r}=\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda(-\mathbf{i}+\mathbf{j}+2\mathbf{k})$, or equivalent correct form	**A1**	e.g. $\mathbf{r}=\begin{pmatrix}0\\3\\1\end{pmatrix}+\mu\begin{pmatrix}1\\-1\\-2\end{pmatrix}$ Allow $\begin{pmatrix}x\\y\\z\end{pmatrix}$ for $\mathbf{r}$.
		3	
10(b)	Use a correct method to find the position vector of C	**M1**	e.g. $\mathbf{OC}=\mathbf{OA}+\mathbf{AC}=\begin{pmatrix}1-3\\2+3\\-1+6\end{pmatrix}$
	Obtain answer $-2\mathbf{i}+5\mathbf{j}+5\mathbf{k}$, or equivalent	**A1**	Accept as coordinates.
		2	

10(c)	State $\overline{OP}$ in component form	**B1 FT**	
	Form an equation in λ by equating the modulus of OP to $\sqrt{14}$, or equivalent	**M1**	
	Simplify and obtain $3\lambda^2-\lambda-4=0$, or equivalent	**A1**	$3\lambda^2+\lambda-4=0$ if using $\mathbf{i}-\mathbf{j}-2\mathbf{k}$ in **(a)**. $3\mu^2+5\mu-2=0$ if using $-\mathbf{i}+\mathbf{j}+2\mathbf{k}$ in **(a)** and OB.
	Solve a 3-term quadratic and find a position vector	**M1**	$\left(\lambda=-1,\frac{4}{3}\text{ or }\lambda=1,-\frac{4}{3}\text{ or }\mu=\frac{1}{3},-2\text{ or }\mu=-\frac{1}{3},2\right)$
	Obtain answers $2\mathbf{i}+\mathbf{j}-3\mathbf{k}$ and $-\frac{1}{3}\mathbf{i}+\frac{10}{3}\mathbf{j}+\frac{5}{3}\mathbf{k}$, or equivalent	**A1**	Accept as coordinates.
		5	

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Q9.

8(a)	State $\overrightarrow{OM} = 4\mathbf{i} + 2\mathbf{j}$	**B1**	
	Use a correct method to find $\overrightarrow{ON}$	**M1**	
	Obtain answer $3\mathbf{j} + \mathbf{k}$	**A1**	
	Use a correct method to find a line equation for *MN*	**M1**	
	Obtain answer $\mathbf{r} = 3\mathbf{j} + \mathbf{k} + \lambda(4\mathbf{i} - \mathbf{j} - \mathbf{k})$, or equivalent	**A1**	
		5	

8(b)	Taking a general point *P* on *MN*, form an equation in λ by *either* equating a relevant scalar product to zero *or* equating the derivative of $\overrightarrow{OP}$ to zero *or* using Pythagoras in triangle *OPM* or *OPN*	**M1**	
	Obtain $\lambda = \frac{2}{9}$	**A1**	OE
	Use correct method to find *OP*	**M1**	
	Obtain the given answer correctly	**A1**	
	Alternative method to Question 8(b)		
	Use a scalar product to find the projection of *OM* (or *ON*) on *MN*	**M1**	
	Obtain answer $\frac{14}{\sqrt{18}} \left(\text{or } \frac{4}{\sqrt{18}} \right)$	**A1**	
	Use Pythagoras to obtain the perpendicular	**M1**	
	Obtain the given answer correctly	**A1**	
		4	

Q10.

10(a)	Obtain direction vector $-\mathbf{i} - 3\mathbf{j} + \mathbf{k}$	**B1**	OE
	Use a correct method to form a vector equation	**M1**	
	Obtain answer $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ or $\mathbf{r} = \mathbf{i}\ 2\mathbf{j} + 2\mathbf{k} + \lambda(-\mathbf{i} - 3\mathbf{j} + \mathbf{k})$	**A1**	Need **r** or r on LHS
		3	
10(b)	Carry out the correct process for evaluating the scalar product of the direction vectors.	**M1**	$(-1, -3, 1) . (1, -3, -2) = -1 + 9 - 2$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse cosine of the result for any 2 vectors	**M1**	$\cos^{-1}\left(\frac{1+9\ 2}{(1+9+1)(1+9+4))}\right)$
	Obtain answer 61.1°	**A1**	61.086°
		3	

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10(c)	Express general point of AB or l in component form, e.g. $(2-\lambda,\ 1-3\lambda,\ 1+\lambda)$ or $(1+\mu,\ 2-3\mu,\ -3-2\mu)$	**B1**	
	Equate at least two pairs of components and solve for λ or for μ	**M1**	
	Obtain a correct answer for λ or μ, e.g. $\lambda = 6,\ \frac{1}{3}$, or $-\frac{14}{9}$; $\mu = -5,\ \frac{2}{3}$ or $-\frac{11}{9}$	**A1**	
	Verify that all three equations are not satisfied, and the lines do not intersect	**A1**	
	Express general point of AB or l in component form, e.g. $(1-\lambda^*,\ -2-3\lambda^*,\ 2+\lambda^*)$ or $(1+\mu^*,\ 2-3\mu^*,\ -3-2\mu^*)$	**4**	