Vectors 2 - Marking Scheme

Q1.

9(a)	Obtain $\overrightarrow{OM} = \mathbf{i}+2\mathbf{j}+2\mathbf{k}$	B1	
	Use a correct method to find $\overrightarrow{MN}$	M1	e.g. $\overrightarrow{MO}+\overrightarrow{OA}+\overrightarrow{AN}$ or $\overrightarrow{MO}+\overrightarrow{ON}$
	Obtain $\overrightarrow{MN} = \mathbf{i}+\mathbf{j}-2\mathbf{k}$	A1	Accept any notation.
		3	
9(b)	Use a correct method to form an equation for MN	M1	Allow without $\mathbf{r} = ...$
	Obtain $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + \lambda(\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	A1 FT	OE e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ Must have $\mathbf{r} = ...$ Follow *their* answers to part **9(a)**.
		2	
9(c)	State $\overrightarrow{OP}$ for a general point P on MN in component form, e.g. $(2+\lambda, 3+\lambda, -2\lambda)$	B1	
	Equate scalar product of $\overrightarrow{OP}$ and a direction vector for MN to zero and solve for λ	M1	
	Obtain $\lambda = -\frac{5}{6}$	A1	OE e.g. $\mu = \frac{1}{6}$
	Obtain $\sqrt{\frac{53}{6}}$ correctly	A1	AG e.g. from $\sqrt{\left(\frac{7}{6}\right)^2+\left(\frac{13}{6}\right)^2+\left(\frac{5}{3}\right)^2}$
		4	

Q2.

9(a)	Express general point of l or m in component form, i.e. $(-1+2\lambda, 3-\lambda, 4-\lambda)$ or $(5+a\mu, 4+b\mu, 3+\mu)$	B1	
	Equate components and eliminate either λ or μ	M1	e.g. $\mu=\frac{2}{1-b}$, $\lambda=\frac{-1-b}{1-b}$, $\mu=\frac{-4}{2+a}$, $\lambda=\frac{a+6}{a+2}$
	Eliminate the other parameter or obtain a second expression in the first	M1	λ and μ are not required to be the subject of the equations.
	Show intermediate steps to obtain $2b-a=4$	A1	AG
	Alternative method for question 9(a)		
	Express general point of l or m in component form, i.e. $(-1+2\lambda, 3-\lambda, 4-\lambda)$ or $(5+a\mu, 4+b\mu, 3+\mu)$	B1	
	Express a or b in terms of λ and μ	M1	$a=\frac{2\lambda-6}{\mu}$, $b=\frac{-1-\lambda}{\mu}$
	Use $\lambda = 1-\mu$	M1	
	Obtain $2b-a=4$	A1	AG
		4	
9(b)	Using the correct process equate the scalar product of the direction vectors to zero	*M1	$(2\mathbf{i}-\mathbf{j}-\mathbf{k}).(a\mathbf{i}+b\mathbf{j}+\mathbf{k})=0$ SOI.
	Obtain $2a-b-1=0$	A1	OE e.g. $2(2b-4)-b-1=0$
	Solve simultaneous equations for a or for b	DM1	
	Obtain $a=2, b=3$	A1	
		4	

Q3.

9(a)	Using the correct process find the scalar product of direction vectors of *l* and **OA**	**M1**	(1, 5, 6).(– 1, 2, 3) = –1.1 + 5.2 + 6.3 = – 1 + 10 + 18
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse cosine of the result	**M1**	*Their* scalar product ÷ $[\sqrt{(1^2+5^2+6^2)}\sqrt{((-1)^2+2^2+3^2)}]$. Angle = $\cos^{-1}\dfrac{27}{\sqrt{62}\sqrt{14}}$.
	Obtain answer 23.6°.	**A1**	AWRT 23.6°. 23.5889°. Radians 0.412 scores A0 (0.4117…).
		3	
9(b)	Taking a general point *P* on *l*, state **AP** (or **PA**) in component form, e.g. $(3-\lambda, -5+2\lambda, -5+3\lambda)$	**B1**	Note: (4, 1, 0) or (4, 1, 1), for **4i + k** is not MR, but M1 possible.
	Either equate scalar product of **AP** and direction vector of *l* to zero and solve for λ *or* use Pythagoras in a relevant triangle and solve for λ	**M1**	$(3-\lambda, -5+2\lambda, -5+3\lambda).(-1, 2, 3) = 0$ $-3\ -10\ -15+\lambda+4\lambda+9\lambda = 0$ or let **OQ** = (4, 0, 1) so **AQ** = (3, – 5, – 5), **QP** = $(-\lambda, 2\lambda, 3\lambda)$, **AP** = $(3-\lambda, -5+2\lambda, -5+3\lambda)$ hence $3^2+(-5)^2+(-5)^2 =$ $(3-\lambda)^2+(-5+2\lambda)^2+(-5+3\lambda)^2+(-\lambda)^2+(2\lambda)^2+(3\lambda)^2$ Other alternative approaches are possible, e.g. minimise *AP* or AP^2, either by completing the square or by differentiating.
	Obtain $\lambda = 2$	**A1**	$\lambda = 2$
	State that the position vector **OP*** of the foot is **2i + 4j + 7k**	**A1**	OE Condone coordinates.
		4	
9(c)	Set up a correct method for finding the position vector of the reflection of *A* in *l*	**M1**	For all methods, allow a sign error in one component only: **OA′** = **OP*** + (**OP*** – **OA**) $their(2,4,7)+(their\,2,4,7-1,5,6)$ or **OA′** = **OP*** – (**OA** – **OP***) $their(2,4,7)-(1,5,6-their\,2,4,7)$ or **OA′** = **OA** + 2(**OP*** – **OA**) $\begin{pmatrix}1+2(their\,2-1)\\5+2(their\,4-5)\\6+2(their\,7-6)\end{pmatrix}$ or midpoint **OP*** = (**OA** + **OA′**)/2 with *their* λ value substituted. $\dfrac{1+x}{2}=their\,2\quad \dfrac{5+y}{2}=their\,4\quad \dfrac{6+z}{2}=their\,7$
	Obtain answer **3i** + **3j** + **8k** or $3\left(\mathbf{i}+\mathbf{j}+\dfrac{8}{3}\right)$	**A1**	OE Condone coordinates $x=3, y=3, z=8$ A1. No method shown and correct answer 2/2.
		2	

WALTER MATHS

Q4.

11(a)	State $\overrightarrow{OM} = 2\mathbf{i} + 2\mathbf{j}$ or equivalent	B1	Can be implied by $\overrightarrow{MB} = -2\mathbf{i} + 2\mathbf{j}$ or $\overrightarrow{MA} = 2\mathbf{i} - 2\mathbf{j}$.
	Obtain $\overrightarrow{MD} = -2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$	B1	
	Use a correct method to find $\overrightarrow{ON}$	M1	e.g. $\overrightarrow{OC} + \frac{2}{3}\overrightarrow{CB}$
	Obtain answer $3\mathbf{j} + \mathbf{k}$	A1	
		4	
11(b)	Use the correct process for evaluating the scalar product of $\overrightarrow{MD}$ and $\overrightarrow{ON}$	M1	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and reach the inverse cosine of the result	M1	$\cos^{-1}\left(\frac{-6+3}{\sqrt{10}\sqrt{17}}\right)$
	Obtain final answer 103.3°	A1	
		3	

11(c)	Taking a general point P of ON to have position vector $\lambda(3\mathbf{j}+\mathbf{k})$, form an equation in λ by *either* equating the scalar product of $\overrightarrow{ON}$ and $\overrightarrow{MP}$ to zero, *or* applying Pythagoras to triangle OMP, *or* equating the derivative of $\lvert\overrightarrow{MP}\rvert$ to zero	M1	e.g. $\begin{pmatrix} -2 \\ -2+3\lambda \\ \lambda \end{pmatrix} . \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} = 0$
	Solve and obtain $\lambda = \frac{3}{5}$	A1	
	Substitute for λ and calculate MP	M1	$\overrightarrow{MP} = -2\mathbf{i} - \frac{1}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$
	Obtain $\sqrt{\frac{22}{5}}$	A1	AG
	Alternative method for question 11(c)		
	Use a scalar product to find the projection OQ of OM on OM	M1	
	Obtain $OQ = \frac{6}{\sqrt{10}}$	A1	
	Use Pythagoras in triangle OMQ to find MQ	M1	
	Obtain $\sqrt{\frac{22}{5}}$	A1	AG
		4	

Q5.

6(a)	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AC}$ correctly in component form	B1	$(\overrightarrow{AB} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k},\ \overrightarrow{AC} = 4\mathbf{i} - 3\mathbf{k})$.
	Using the correct process with relevant vectors to evaluate the scalar product $\overrightarrow{AB}.\overrightarrow{AC}$,	M1	or $\overrightarrow{BA}.\overrightarrow{CA}$ $(8-3=5)$. M0 for $\overrightarrow{AB}.\overrightarrow{CA}$.
	Using the correct process for the moduli, divide *their* scalar product by the product of *their* moduli to obtain $\cos\theta$ or θ	M1	$\left(\frac{5}{\sqrt{9}\sqrt{25}}\right)$ Independent of the first M1.
	Obtain answer $\frac{1}{3}$	A1	ISW. Need to see a value for $\cos\theta$. Accept $\frac{5}{15}$ or 0.333 ($\cos^{-1}\frac{1}{3}$ alone is not sufficient)
		4	

6(b)	Use correct method to find an **exact** value for the sine of angle *BAC* from *their* **(a)**	M1	$\left(\sqrt{1-\frac{1}{9}}\right)$
	Obtain answer $\frac{2}{3}\sqrt{2}$, or equivalent	A1	
	Use correct area formula to find the area of triangle *ABC* with *their* versions of relevant vectors	M1	$\left(\frac{1}{2}\sqrt{9}\sqrt{25}\times \textit{their}\ \sin\theta\right)$ or $\frac{1}{2}\sqrt{9}\sqrt{25}\times\sin\left(\cos^{-1}\frac{1}{3}\right)$
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	A1	Only ISW
	Alternative method 1 for question 6(b)		
	Use correct method to find the perpendicular distance from *A* to *BC* (or *B* to *AC* or *C* to *AB*)	M1	$\begin{pmatrix}2+2\lambda\\-2+2\lambda\\1-4\lambda\end{pmatrix}.\begin{pmatrix}2\\2\\-4\end{pmatrix}=0\Rightarrow\lambda=\frac{1}{6}$
	Obtain $\frac{1}{3}\sqrt{75}$	A1	$\left(\left\vert\frac{7}{3}\mathbf{i}-\frac{5}{3}\mathbf{j}+\frac{1}{3}\mathbf{k}\right\vert\right)$
	Use correct area formula to find the area of triangle *ABC*	M1	$\left(\frac{1}{2}\times \textit{their}\sqrt{24}\times \textit{their}\ \frac{1}{3}\sqrt{75}\right)$ The length they use for *their* base must be found correctly.
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	A1	

6(b)	**Alternative method 2 for question 6(b)**		
	Correct method to find the semi-perimeter	M1	
	Obtain $4+\sqrt{6}$	A1	
	Correct application of Hero's (Heron's) formula	M1	$\sqrt{\left(4+\sqrt{6}\right)\left(1+\sqrt{6}\right)\left(-1+\sqrt{6}\right)\left(4-\sqrt{6}\right)}$
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	A1	
		4	

Q6.

9(a)	State $\overrightarrow{OM}=\begin{pmatrix}2\\1\\0\end{pmatrix}$	B1	
	Use a correct method to find $\overrightarrow{ON}$	M1	
	Obtain answer $\begin{pmatrix}3\\-2\\-1\end{pmatrix}$	A1	
		3	
9(b)	Carry out a correct method to form a vector equation for *MN*	M1	
	Obtain a correct equation in any form, e.g. $\mathbf{r}=\begin{pmatrix}2\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-3\\-1\end{pmatrix}$	A1	OE
		2	

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9(c)	State a correct vector equation for AB in any form, e.g. $\mathbf{r} = \begin{pmatrix}1\\0\\1\end{pmatrix} + \mu\begin{pmatrix}-1\\5\\1\end{pmatrix}$	B1	
	Equate components of AB and MN and solve for λ or for μ	M1	
	Obtain $\lambda = -3$ or $\mu = 2$	A1	
	Obtain position vector $\begin{pmatrix}-1\\10\\3\end{pmatrix}$, or equivalent, for Q	A1	
		4	

Q7.

10(a)	Carry out correct process for evaluating the scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OB}$	M1	$\pm(3, -1, 2).(1, 2, -3) = \pm(3-2-6) = [-5]$.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and obtain $\cos^{-1}\{\pm(3-2-6)/[\sqrt{(3^2+(-1)^2+2^2)}\ \sqrt{(1^2+2^2+(-3)^2)}]\}$	A1	
	Obtain answer 110.9° or 1.94^c	A1	
		3	
10(b)	Use a correct method to form an equation for line through AB	M1	
	Obtain $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \mu_1(2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$	A1	OE e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \mu_2(-2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k})$. Need $\mathbf{r}$ or (x, y, z).
		2	
10(c)	Obtain a correct equation for line through CD e.g. $[\mathbf{r} =]\ \mathbf{i} - 2\mathbf{j} + 5\mathbf{k} + \lambda_1(-4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$	B1	OE e.g. $[\mathbf{r} =]\ 5\mathbf{i} - 6\mathbf{j} + 11\mathbf{k} + \lambda_2(-4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$. $\mathbf{r}$ can be omitted or another symbol used.
	Equate two pairs of components of general points on *their l* and *their CD* and solve for λ or for μ	M1	
	Obtain e.g. $\lambda_1 = -2$ or $\mu_1 = 3$ or $\lambda_2 = -1$ or $\mu_2 = -4$	A1	
	Obtain position vector $9\mathbf{i} - 10\mathbf{j} + 17\mathbf{k}$	A1	Condone (9, –10, 17) but not ($9\mathbf{i}$, $-10\mathbf{j}$, $17\mathbf{k}$).
		4	

Q8.

6(a)	Obtain a vector for one side of the parallelogram	B1	e.g. $\overrightarrow{AB} = \begin{pmatrix}2\\2\\-1\end{pmatrix}$ or $\overrightarrow{BC} = \begin{pmatrix}-1\\-5\\-6\end{pmatrix}$.
	Correct method to obtain $\pm\overrightarrow{OD}$	M1	e.g. $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC}$. MO if use $\overrightarrow{AB} = \overrightarrow{CD}$ or $\overrightarrow{BC} = \overrightarrow{DA}$.
	Obtain $\overrightarrow{OD} = \mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$	A1	Any equivalent form. Accept coordinates.
		3	
6(b)	Using the correct process, evaluate the scalar product $\overrightarrow{BA}.\overrightarrow{BC}$	M1	$(2+10-6)$ Scalar product of two relevant vectors. OE
	Using the correct process for the moduli, divide the scalar product by the product of the moduli.	M1	$\dfrac{2+10-6}{\sqrt{9}\times\sqrt{62}}$.
	Obtain answer $\dfrac{2}{\sqrt{62}}$	A1	ISW Or simplified equivalent i.e. $\dfrac{\sqrt{62}}{31}$.
		3	

6(c)	State or imply $\sin\theta = \sqrt{\frac{58}{62}}$	B1 FT	Follow *their* $\cos\theta$.
	Use correct method to find the area of *ABCD*	M1	**e.g.** $2\times\frac{1}{2}BA\times BC\sin\theta$. Condone decimals.
	Correct unsimplified expression for the area	A1 FT	**e.g.** $2\times\frac{1}{2}\times 3\times\sqrt{62}\times\sin\theta$. Condone decimals. Follow *their* sides and angle.
	Obtain answer $3\sqrt{58}$	A1	Correct only.
		4	

Q9.

11(a)	Carry out correct method for finding a vector equation for *AB*	M1	
	Obtain [**r** =] **i** + 2**j** – 2**k** + λ (**i** – 3**j** + 3**k**)	A1	OE e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$.
	Equate two pairs of components of general points on *their AB* and *l* and evaluate λ or μ	M1	$\begin{pmatrix} 1+\lambda \\ 2-3\lambda \\ -2+3\lambda \end{pmatrix} = \begin{pmatrix} 1+2\mu \\ -1-3\mu \\ 3+4\mu \end{pmatrix}$.
	Obtain correct answer for λ or μ, e.g. $\lambda = -1$, $\mu = -2$	A1	Correct value from two correct component equations.
	Verify that all three equations are not satisfied and the lines fail to intersect ($\neq$ is sufficient justification e.g. $0 \neq -3$).	A1	Conclusion needs to follow correct values. Hybrid versions are possible e.g. using **j** and **k** to get one parameter and then **i** to obtain the other. or e.g. solving two pairs of simultaneous equations and showing that the results are not the same. Alternatives: (see table below)
		5	

A	λ	μ		*B*	λ	μ	
ij	2	1	$4 \neq 7$	**ij**	1	1	$4 \neq 7$
ik	5	5/2	$-13 \neq -17/2$	**ik**	4	5/2	$-13 \neq -17/2$
jk	–1	–2	$0 \neq -3$	**jk**	–2	–2	$0 \neq -3$

11(b)	Find $\overrightarrow{AP}$ for a general point *P* on *l*, e.g. –3**j** + 5**k** + μ(2**i** – 3**j** + 4**k**)	B1	Or equivalent e.g. $\overrightarrow{PA} = -2\mu\mathbf{i} + (3\mu+3)\mathbf{j} - (4\mu+5)\mathbf{k}$.
	Calculate scalar product of *their* $\overrightarrow{AP}$ and a direction vector for *l* and equate the result to zero	M1	e.g. $4\mu + (9+9\mu) + (20+16\mu) = 0$. M0 if using $\overrightarrow{OP}$. M0 if using parallel line through *A*.
	Obtain $\mu = -1$	A1	
	Obtain answer –**i** + 2**j** – **k**	A1	Accept coordinates in place of position vector.
	Alternative Method for Question 11(b)		
	Find $\overrightarrow{AP}$ for a general point *P* on *l*, e.g. –3**j** + 5**k** + μ(2**i** – 3**j** + 4**k**)	B1	Or equivalent e.g. $\overrightarrow{PA} = -2\mu\mathbf{i} + (3\mu+3)\mathbf{j} - (4\mu+5)\mathbf{k}$.
	Use Pythagoras and differentiate with respect to μ to obtain value of μ corresponding to minimum distance. (No need to prove it is a minimum)	M1	$\frac{\mathrm{d}}{\mathrm{d}\mu}\left(4\mu^2 + 9(\mu+1)^2 + (4\mu+5)^2\right) = 0$.
	Obtain $\mu = -1$	A1	
	Obtain answer –**i** + 2**j** – **k**	A1	Accept coordinates in place of position vector.
		4	

Q10.

9(a)	Perform scalar product of direction vectors and set result equal to zero	**M1**	$2c + 6 + 4 = 0$.
	Use P to find the value of λ	**M1**	$3 - 2\lambda = 7 \Rightarrow \lambda = -2$ [$a + \lambda c = 4$, $b + 4\lambda = -2$]. Equation for line l may contain $-\lambda$ instead of $+\lambda$ leading to $\lambda = 2$ all marks available.
	Obtain $c = -5$ or $b = 6$	**A1**	
	$a = -6$, $b = 6$ and $c = -5$ all correct	**A1**	
		4	**SC1**: Use P to find the value of λ M1 Substitute $\lambda = -2$ into point P, so $a - 2c = 4$, and put $\mu = -1$ and $\lambda = -1$ into l so $a - c = -1$, then solve to obtain $a = -6$, $b = 6$ and $c = -5$. All 3 values correct A1. Max 2/4.

9(b)	Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on m $= \pm((1 + 2\mu, 2 - 3\mu, 3 + \mu) - (a + \lambda c, 3 - 2\lambda, b + 4\lambda))$	**B1**	$\left[\overrightarrow{PQ} \text{ or } \overrightarrow{QP} = \pm\begin{pmatrix}-3+2\mu\\-5-3\mu\\5+\mu\end{pmatrix}\right]$ Could be *their* a, b, c and λ values provided M1 M1 gained in **(a)**. Allow expression in answer column.
	Equate the scalar product of $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) and a direction vector for m to zero and obtain an equation in μ	**M1***	$(2(-3+2\mu) - 3(-5-3\mu) + (5+\mu)) = 0$. Allow $\overrightarrow{PQ} = \overrightarrow{OQ} + \overrightarrow{OP}$ sign problem.
	Solve and obtain $\mu = -1$	**A1**	$PQ^2 = (-3+2\mu)^2 + (-5-3\mu)^2 + (5+\mu)^2$. [$= 14(\mu + 1)^2 + 45$]. Min when $\mu = -1$ or by differentiation.
	Obtain $\overrightarrow{OQ} = -\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ or $\overrightarrow{PQ} = -5\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ Must be labelled correctly	**A1**	The working may be in **(a)** provided at least this result is used in **(b)**.
	Carry out a method to find the position vector of R **Alternative method for DM1** $\overrightarrow{OR} = (4, 7, -2) + t(-5, -2, 4)$ $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$ Solve $\lvert \mathbf{QR} \rvert^2 = \frac{9}{4} \lvert \mathbf{PQ} \rvert^2$ or $\lvert \mathbf{QR} \rvert = \frac{3}{2} \lvert \mathbf{PQ} \rvert$ $t = 2.5$	**DM1**	e.g. Use $\overrightarrow{OR} = \overrightarrow{OP} + \frac{5}{2}\overrightarrow{PQ}$ or $\overrightarrow{OR} = \overrightarrow{OQ} + \frac{3}{2}\overrightarrow{PQ}$ or $\overrightarrow{OR} = \frac{5}{2}\overrightarrow{OQ} - \frac{3}{2}\overrightarrow{OP}$ or $2\overrightarrow{QR} = 2(\overrightarrow{OR} - \overrightarrow{OQ}) = 3\overrightarrow{PQ}$ where $\overrightarrow{OR} = (x, y, z)$. $\overrightarrow{PQ}$ used in all these approaches, may be incorrect, must be in the correct direction, i.e. not using $\overrightarrow{QP}$ for $\overrightarrow{PQ}$.