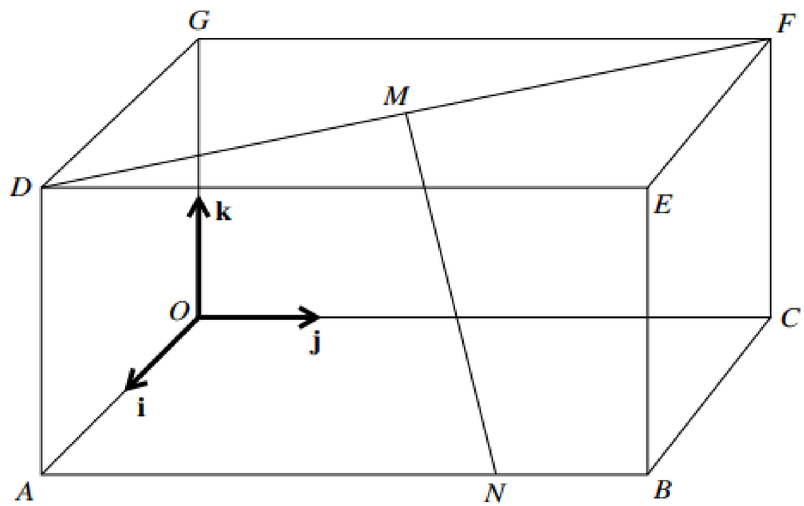
Vectors 2

Q1.



In the diagram, $OABCDEFG$ is a cuboid in which $OA = 2$ units, $OC = 4$ units and $OG = 2$ units. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OC and OG respectively. The point M is the midpoint of DF. The point N on AB is such that $AN = 3NB$.

(a) Express the vectors $\overrightarrow{OM}$ and $\overrightarrow{MN}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(b) Find a vector equation for the line through M and N. [2]

(c) Show that the length of the perpendicular from O to the line through M and N is $\sqrt{\frac{53}{6}}$. [4]

Q2.

The lines l and m have vector equations

$$\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} + \mathbf{k})$$

respectively, where a and b are constants.

(a) Given that l and m intersect, show that $2b - a = 4$. [4]

(b) Given also that l and m are perpendicular, find the values of a and b. [4]

(c) When a and b have these values, find the position vector of the point of intersection of l and m. [2]

Q3.

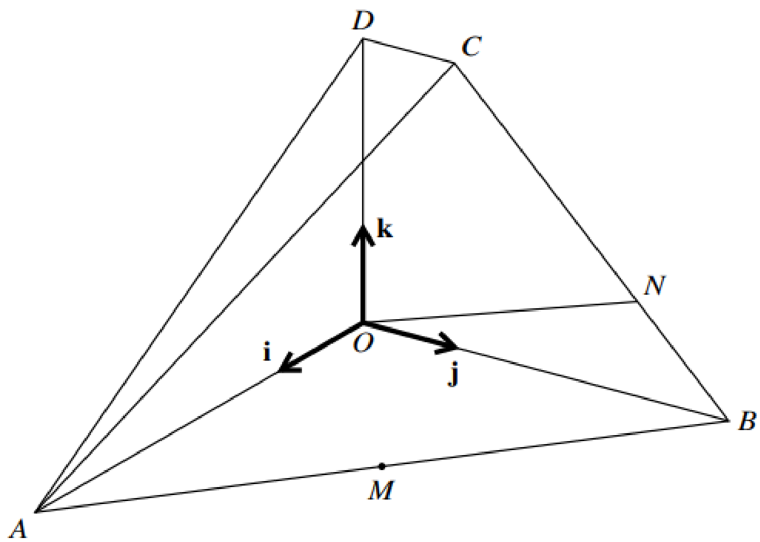
With respect to the origin O, the point A has position vector given by $\overrightarrow{OA} = \mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$. The line l has vector equation $\mathbf{r} = 4\mathbf{i} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

(a) Find in degrees the acute angle between the directions of OA and l. [3]

(b) Find the position vector of the foot of the perpendicular from A to l. [4]

(c) Hence find the position vector of the reflection of A in l. [2]

Q4.



In the diagram, $OABCD$ is a solid figure in which $OA = OB = 4$ units and $OD = 3$ units. The edge OD is vertical, DC is parallel to OB and $DC = 1$ unit. The base, OAB, is horizontal and angle $AOB = 90°$. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OB and OD respectively. The midpoint of AB is M and the point N on BC is such that $CN = 2NB$.

(a) Express vectors $\overrightarrow{MD}$ and $\overrightarrow{ON}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [4]

(b) Calculate the angle in degrees between the directions of $\overrightarrow{MD}$ and $\overrightarrow{ON}$. [3]

(c) Show that the length of the perpendicular from M to ON is $\sqrt{\frac{22}{5}}$. [4]

Q5.

Relative to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}.$$

(a) Using a scalar product, find the cosine of angle BAC. [4]

(b) Hence find the area of triangle ABC. Give your answer in a simplified exact form. [4]

Q6.

With respect to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}.$$

The midpoint of AC is M and the point N lies on BC, between B and C, and is such that $BN = 2NC$.

(a) Find the position vectors of M and N. [3]

(b) Find a vector equation for the line through M and N. [2]

(c) Find the position vector of the point Q where the line through M and N intersects the line through A and B. [4]

Q7.

With respect to the origin O, the points A, B, C and D have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 5 \\ -6 \\ 11 \end{pmatrix}.$$

(a) Find the obtuse angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. [3]

The line l passes through the points A and B.

(b) Find a vector equation for the line l. [2]

(c) Find the position vector of the point of intersection of the line l and the line passing through C and D. [4]

Q8.

Relative to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \qquad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}.$$

The quadrilateral $ABCD$ is a parallelogram.

(a) Find the position vector of D. [3]

(b) The angle between BA and BC is θ.

Find the exact value of $\cos\theta$. [3]

(c) Hence find the area of $ABCD$, giving your answer in the form $p\sqrt{q}$, where p and q are integers. [4]

Q9.

The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$.

(a) Show that l does not intersect the line passing through A and B. [5]

(b) Find the position vector of the foot of the perpendicular from A to l. [4]

Q10.

The lines l and m have equations

$$l: \quad \mathbf{r} = a\mathbf{i} + 3\mathbf{j} + b\mathbf{k} + \lambda(c\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}),$$
$$m: \quad \mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}).$$

Relative to the origin O, the position vector of the point P is $4\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$.

(a) Given that l is perpendicular to m and that P lies on l, find the values of the constants a, b and c. [4]

(b) The perpendicular from P meets line m at Q. The point R lies on PQ extended, with $PQ : QR = 2 : 3$.

Find the position vector of R. [6]
