

# Rational Functions and Graphs 1



Q1 The curve  $C$  has equation

$$y = \frac{(x-2)(x-a)}{(x-1)(x-3)},$$

where  $a$  is a constant not equal to 1, 2 or 3.

(i) Write down the equations of the asymptotes of  $C$ . [2]

(ii) Show that  $C$  meets the asymptote parallel to the  $x$ -axis at the point where  $x = \frac{2a-3}{a-2}$ . [2]

(iii) Show that the  $x$ -coordinates of any stationary points on  $C$  satisfy

$$(a-2)x^2 + (6-4a)x + (5a-6) = 0,$$

and hence find the set of values of  $a$  for which  $C$  has stationary points. [6]

(iv) Sketch the graph of  $C$  for

(a)  $a > 3$ ,

(b)  $2 < a < 3$ .

[4]

Q2 The curve  $C$  has equation

$$y = \frac{x^2 - 5x + 4}{x + 1}.$$

(i) Obtain the coordinates of the points of intersection of  $C$  with the axes. [2]

(ii) Obtain the equation of each of the asymptotes of  $C$ . [3]

(iii) Draw a sketch of  $C$ . [3]

Q3 The curve  $C$  has equation

$$y = \frac{x^2 - 3x - 7}{x + 1}.$$

(i) Obtain the equations of the asymptotes of  $C$ . [3]

(ii) Show that  $\frac{dy}{dx} > 1$  at all points of  $C$ . [2]

(iii) Draw a sketch of  $C$ . [3]

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Q4 The curve  $C$  has equation

$$y = \frac{x(x+1)}{(x-1)^2}.$$

- (i) Obtain the equations of the asymptotes of  $C$ . [3]
- (ii) Show that there is exactly one point of intersection of  $C$  with the asymptotes and find its coordinates. [2]
- (iii) Find  $\frac{dy}{dx}$  and hence
- (a) find the coordinates of any stationary points of  $C$ ,
- (b) state the set of values of  $x$  for which the gradient of  $C$  is negative. [6]
- (iv) Draw a sketch of  $C$ . [3]
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Q5 The curve  $C$  with equation

$$y = \frac{ax^2 + bx + c}{x-1},$$

where  $a$ ,  $b$  and  $c$  are constants, has two asymptotes. It is given that  $y = 2x - 5$  is one of these asymptotes.

- (i) State the equation of the other asymptote. [1]
- (ii) Find the value of  $a$  and show that  $b = -7$ . [3]
- (iii) Given also that  $C$  has a turning point when  $x = 2$ , find the value of  $c$ . [3]
- (iv) Find the set of values of  $k$  for which the line  $y = k$  does not intersect  $C$ . [4]
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Q6 The curve  $C$  has equation  $y = \frac{x^2 + px + 1}{x-2}$ , where  $p$  is a constant. Given that  $C$  has two asymptotes, find the equation of each asymptote. [3]

Find the set of values of  $p$  for which  $C$  has two distinct turning points. [5]

Sketch  $C$  in the case  $p = -1$ . Your sketch should indicate the coordinates of any intersections with the axes, but need not show the coordinates of any turning points. [3]

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