

1 The roots of the equation

$$x^3 + px^2 + qx + r = 0$$

are $\alpha, 2\alpha, 4\alpha$, where p, q, r and α are non-zero real constants.

(i) Show that

$$2p\alpha + q = 0. \quad [4]$$

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(ii) Show that

$$p^3r - q^3 = 0. \quad [2]$$

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2 The roots of the cubic equation $x^3 + 2x^2 - 3 = 0$ are α , β and γ .

(i) By using the substitution $y = \frac{1}{x^2}$, find the cubic equation with roots $\frac{1}{\alpha^2}$, $\frac{1}{\beta^2}$ and $\frac{1}{\gamma^2}$. [3]

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(ii) Hence find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$. [1]

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(iii) Find also the value of $\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2}$. [1]

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3 The cubic equation $2x^3 - 3x^2 + 4x - 10 = 0$ has roots α , β and γ .

(i) Find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. [4]

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(ii) Find the value of $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$. [4]

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(iii) Find the value of S_{-2} .

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