

Q1.

2(i)	$\alpha + 2\alpha + 4\alpha = -p$	B1	Sum of roots.
	$2\alpha^2 + 4\alpha^2 + 8\alpha^2 = q$	B1	Sum of products in pairs.
	$\frac{14\alpha^2}{7\alpha} = -\frac{q}{p}$	M1	Combines equations.
	$\Rightarrow 2p\alpha + q = 0$	A1	Verifies result (AG).
		4	
2(ii)	$8\alpha^3 = -r$	B1	Product of roots.
	$\Rightarrow r = \frac{q^3}{p^3} \Rightarrow p^3r - q^3 = 0$	B1	Verifies result (AG).
		2	

Q2.

1(i)	$y = \frac{1}{x^2} \Rightarrow x = \frac{1}{\sqrt{y}}$	M1	Rearranging to make x the subject
	$\frac{1}{y\sqrt{y}} + \frac{2}{y} - 3 = 0 \Rightarrow \frac{1}{y\sqrt{y}} = 3 - \frac{2}{y}$	M1	Substituting and squaring
	$\Rightarrow 9y^3 - 12y^2 + 4y - 1 = 0$ SR B1 for finding cubic by manipulating roots	A1	OE
	Total:	3	
1(ii)	$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{12}{9}$ or $\frac{4}{3}$	B1FT	
	Total:	1	
1(iii)	$\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2} = \frac{4}{9}$	B1FT	
	Total:	1	

Q3.

4(i)	$\alpha + \beta + \gamma = \frac{3}{2} \quad \alpha\beta + \beta\gamma + \gamma\alpha = 2 \quad \alpha\beta\gamma = 5 + \beta + \gamma =$ $\frac{3}{2}\alpha\beta + \beta\gamma + \gamma\alpha = 2\alpha\beta\gamma = 5$	B1	(Can be awarded in (ii) if not seen here) SOI
	$(\alpha + 1)(\beta + 1)(\gamma + 1) = \alpha\beta\gamma + (\alpha\beta + \beta\gamma + \gamma\alpha) +$ $(\alpha + \beta + \gamma) + 1$	M1A1	Multiply out and group for M1
	$= 5 + 2 + 1\frac{1}{2} + 1 = 9\frac{1}{2}$	A1FT	Alt method: Let $x = y - 1$ M1 Sub and expand $2y^3 - 9y^2 - 16y - 19 = 0$ M1, A1 Product of roots = $19/2$ A1
		4	
4(ii)	$(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta) = \left(\frac{1}{2} - \alpha\right)\left(\frac{1}{2} - \beta\right)\left(\frac{1}{2} - \gamma\right)$	M1	Alt methods: $= (\Sigma\alpha)(\Sigma\alpha\beta) - \alpha\beta\gamma$ or $\Sigma\alpha^2\Sigma\alpha + 2\alpha\beta\gamma - \Sigma\alpha^3$
	$= \frac{27}{8} - \frac{9}{4}(\alpha + \beta + \gamma) + \frac{3}{2}(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma$	A1	
	$= \frac{27}{8} - \frac{9}{4} \times \frac{3}{2} + \frac{3}{2} \times 2 - 5 = -2$	M1A1	
		4	

Q4.

6(i)	Substitutes $x = \frac{y+1}{3}$	M1	Accept substitution of $y = 3x - 1$ into given equation and derivation of equation in x .
	Obtains the given result	A1	AG.
6(ii)	$S_3 = 2S_1 + 7 \times 3$	M1	Uses $y^3 = 2y + 7$. Or uses formula for $\Sigma(3\alpha - 1)^3$
	$= 21$	A1	
6(iii)	$S_{-1} = \frac{(3\alpha - 1)(3\beta - 1) + (3\alpha - 1)(3\gamma - 1) + (3\beta - 1)(3\gamma - 1)}{(3\alpha - 1)(3\beta - 1)(3\gamma - 1)} = \frac{-2}{7}$	M1 A1	Award M1A1 if $S_{-1} = -\frac{2}{7}$ written down directly.
	$7S_{-2} = S_1 - 2S_{-1}$	M1	Uses $7y^{-2} = y - 2y^{-1}$.
	$s_{-2} = \frac{4}{49}$	A1	
	Alt method: $S_{-2} = \Sigma \frac{1}{(3\alpha - 1)^2} = \frac{\Sigma(3\alpha - 1)^2(3\beta - 1)^2}{(3\alpha - 1)^2(3\beta - 1)^2(3\gamma - 1)^2} =$	M1 A1	Alt method: Finds cubic with roots $\frac{1}{3\alpha - 1}$, etc. M1 $7z^3 + 2z^2 - 1 = 0$ A1 Uses $S_2 = (S_1)^2 - 2\Sigma\alpha\beta$ M1 $= \frac{4}{49}$ A1
	$\frac{(\Sigma(3\alpha - 1)(3\beta - 1))^2 - 2(3\alpha - 1)(3\beta - 1)(\Sigma(3\alpha - 1))}{(3\alpha - 1)^2(3\beta - 1)^2(3\gamma - 1)^2}$	M1	
	$= \frac{(-2)2 - 2(7)(0)}{7^2} = \frac{4}{49}$	A1	
		8	