Summation of Series 2

1 Use the method of differences to find $\sum_{r=1}^{n} \frac{1}{(2r)^2 - 1}$. [4]

Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{(2r)^2 - 1}$. [1]

---

2 **(i)** Verify that $\frac{2r+1}{r(r+1)(r+2)} = \frac{1}{2}\left\{\frac{(2r+1)(2r+3)}{(r+1)(r+2)} - \frac{(2r-1)(2r+1)}{r(r+1)}\right\}$. [2]

(ii) Hence show that $\sum_{r=1}^{n} \frac{2r+1}{r(r+1)(r+2)} = \frac{1}{2}\left\{\frac{(2n+1)(2n+3)}{(n+1)(n+2)} - \frac{3}{2}\right\}$. [2]

(iii) Deduce the value of $\sum_{r=1}^{\infty} \frac{2r+1}{r(r+1)(r+2)}$. [2]

---

3 Find $\sum_{r=1}^{n}(4r-3)(4r+1)$, giving your answer in its simplest form. [4]

---

4 **(i)** Verify that

$$\frac{n(\mathrm{e}-1)+\mathrm{e}}{n(n+1)\mathrm{e}^{n+1}} = \frac{1}{n\mathrm{e}^n} - \frac{1}{(n+1)\mathrm{e}^{n+1}}.$$ [1]

Let $S_N = \sum_{n=1}^{N} \frac{n(\mathrm{e}-1)+\mathrm{e}}{n(n+1)\mathrm{e}^{n+1}}$.

(ii) Express S_N in terms of N and e. [2]

Let $S = \lim_{N\to\infty} S_N$.

(iii) Find the least value of N such that $(N+1)(S - S_N) < 10^{-3}$. [3]

Summation of Series 2

5 Let $S_n = \sum_{r=1}^{n} (-1)^{r-1} r^2$.

(i) Use the standard result for $\sum_{r=1}^{n} r^2$ given in the List of Formulae (MF10) to show that

$$S_{2n} = -n(2n+1).$$ [4]

(ii) State the value of $\lim_{n\to\infty} \frac{S_{2n}}{n^2}$ and find $\lim_{n\to\infty} \frac{S_{2n+1}}{n^2}$. [4]

---

6 Let

$$S_N = \sum_{r=1}^{N} (3r+1)(3r+4) \quad \text{and} \quad T_N = \sum_{r=1}^{N} \frac{1}{(3r+1)(3r+4)}.$$

(i) Use standard results from the List of Formulae (MF10) to show that

$$S_N = N(3N^2 + 12N + 13).$$ [3]

(ii) Use the method of differences to show that

$$T_N = \frac{1}{12} - \frac{1}{3(3N+4)}.$$ [3]

(iii) Deduce that $\frac{S_N}{T_N}$ is an integer. [2]

(iv) Find $\lim_{N\to\infty} \frac{S_N}{N^3 T_N}$. [2]

---

7 **(i)** Use the method of differences to show that $\sum_{r=1}^{N} \frac{1}{(3r+1)(3r-2)} = \frac{1}{3} - \frac{1}{3(3N+1)}$. [4]

(ii) Find the limit, as $N \to \infty$, of $\sum_{r=N+1}^{N^2} \frac{N}{(3r+1)(3r-2)}$. [4]
