Summation of Series 2 - MS

Q1.

1	$\frac{1}{(2r-1)(2r+1)} = \frac{1}{2}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)$	M1A1	
	$\sum_{r=1}^{n} \frac{1}{(2r)^2-1} = \frac{1}{2}\left(\left[1-\frac{1}{3}\right]+\left[\frac{1}{3}-\frac{1}{5}\right]+\ldots+\left[\frac{1}{2n-1}-\frac{1}{2n+1}\right]\right) = \frac{1}{2}\left(1-\frac{1}{2n+1}\right)$ (OE)	M1A1	[4]
	$\frac{1}{2}\left(1-\frac{1}{2n+1}\right) = \frac{n}{2n+1} \Rightarrow \sum_{r=1}^{\infty} \frac{1}{(2r)^2-1} = \frac{1}{2}$	B1√	[1]

Q2.

2(i)	RHS $= \frac{1}{2}\left\{\frac{4r^3+8r^2+3r-(4r^2-1)(r+2)}{r(r+1)(r+2)}\right\}$	M1	
	$= \frac{1}{2}\left\{\frac{4r^3+8r^2+3r-(4r^3+8r^2-r-2)}{r(r+1)(r+2)}\right\} = \frac{1}{2}\left\{\frac{(4r+2)}{r(r+1)(r+2)}\right\} = \frac{(2r+1)}{r(r+1)(r+2)}$	A1	AG
	Total:	2	
2(ii)	Sum to n terms is: $\frac{1}{2}\left\{\left[\frac{3.5}{2.3}-\frac{1.3}{1.2}\right]+\left[\frac{5.7}{3.4}-\frac{3.5}{2.3}\right]+\ldots+\left[\frac{(2n-1)(2n+1)}{n(n+1)}-\frac{(2n-3)(2n-1)}{n(n-1)}\right]+\left[\frac{(2n+1)(2n+3)}{(n+1)(n+2)}-\frac{(2n-1)(2n+1)}{n(n+1)}\right]\right\}$	M1	
	$= \frac{1}{2}\left\{\frac{(2n+1)(2n+3)}{(n+1)(n+2)} - \frac{3}{2}\right\}$	A1	AG
2(iii)	$S_\infty = \frac{1}{2}\times 4 - \frac{3}{4} = 1\frac{1}{4}$.	M1A1	
	Total:	4	

Q3.

1	$\sum_{r=1}^{n} u_r = 16\sum_{r=1}^{n} r^2 - 8\sum_{r=1}^{n} r - 3n$	M1A1	**M1** for split into 3 parts
	$= 16\frac{n(n+1)(2n+1)}{6} - 8\frac{n(n+1)}{2} - 3n$	M1	For using formulae correctly in their expression
	$= \ldots = \frac{n}{3}(16n^2+12n-13)$ (3 terms)	A1	OE
		4	

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Q4.

2(i)	$\frac{1}{ne^n}-\frac{1}{(n+1)e^{n+1}}=\frac{(n+1)e-n}{n(n+1)e^{n+1}}=\frac{n(e-1)+e}{n(n+1)e^{n+1}}$	B1	Verifies result (AG).
2(ii)	$S_N=\sum_{n=1}^{N}\left(\frac{1}{ne^n}-\frac{1}{(n+1)e^{n+1}}\right)=\left(\frac{1}{e}-\frac{1}{2e^2}+\frac{1}{2e^2}-\frac{1}{3e^3}+\right.$ ………… $\left.\frac{1}{Ne^N}-\frac{1}{(N+1)e^{N+1}}\right)$ SOI =	M1	Uses difference method to sum.
	$\frac{1}{e}-\frac{1}{(N+1)e^{N+1}}$.	A1	
2(iii)	$S=\frac{1}{e}$	B1	Finds S
	$(N+1)(S-S_N)$ $<10^{-3}\Rightarrow\frac{1}{e^{N+1}}<10^{-3}$	M1	Attempts to find difference between sum and sum to infinity.
	$\Rightarrow e^{N+1}>10^3$ $\Rightarrow$ least such N is 6.	A1	
		6	

Q5.

5(i)	$S_{2n}=1^2-2^2+3^2-4^2$…..	M1	Uses correct difference.
	so $S_{2n}=\sum_{r=1}^{2n}r^2-2\sum_{r=1}^{n}(2r)^2=\sum_{r=1}^{2n}r^2-8\sum_{r=1}^{n}r^2$	A1	Alt method: Use $\sum_1^n(2r-1)^2-\sum_1^n(2r)^2$ =A1
	Thus $S_{2n}=\frac{1}{6}(2n)(2n+1)(4n+1)-\frac{8}{6}n(n+1)(2n+1)$	M1	$\sum_1^n 4r^2-4\sum_1^n(r)+n-4\sum_1^n(r)^2$ M1
	Factorising, $S_{2n}=\frac{1}{3}n(2n+1)(4n+1-4n-4)=-n(2n+1)$	A1	$=-n(2n+1)$ A1 AG
		4	
5(ii)	$\lim_{n\to\infty}\frac{S_{2n}}{n^2}=-2$	B1	
	$S_{2n+1}=S_{2n}+(-1)^{2n}(2n+1)^2$	M1	
	So, $S_{2n+1}=-n(2n+1)+(2n+1)^2=(2n+1)(n+1)$	M1	Uses the result given in **(i)** or using $\lim_{n\to\infty}\frac{S_{2n}}{n^2}$ and correct sign
	Thus $\lim_{n\to\infty}\frac{S_{2n+1}}{n^2}=2$	A1	Alt: Find limit from previous line directly
		4	

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Q6.

7(i)	$\sum_{r=1}^{N}(3r+1)(3r+4)=9\sum_{r=1}^{N}r^2+15\sum_{r=1}^{N}r+4N$	M1	Expands
	$9\left(\frac{1}{6}N(N+1)(2N+1)\right)+15\left(\frac{1}{2}N(N+1)\right)+4N$	M1	Substitutes formulae for $\sum r$ and $\sum r^2$.
	$=N\left(\frac{9}{6}(2N^2+3N+1)+\frac{15}{2}N+\frac{15}{2}+4\right)$ $=N(3N^2+12N+13)$	A1	Shows simplification to the given answer (AG).
		3	
7(ii)	$\frac{1}{(3r+1)(3r+4)}=\frac{1}{3}\left(\frac{1}{3r+1}-\frac{1}{3r+4}\right)$	B1	Finds partial fractions.
	$T_N=\frac{1}{3}\left(\frac{1}{4}-\frac{1}{7}+\frac{1}{7}-\frac{1}{10}+\cdots+\frac{1}{3(N-1)+1}-\frac{1}{3N+4}\right)$	M1	Expresses terms as differences.
	$\frac{1}{3}\left(\frac{1}{4}-\frac{1}{3N+4}\right)=\frac{1}{12}-\frac{1}{3(3N+4)}$	A1	Cancels to given answer (AG).
		3	
7(iii)	$T_N=\frac{N}{4(3N+4)}\Rightarrow\ \frac{S_N}{T_N}=4(3N+4)(3N^2+12N+13)$	M1	Writes $\frac{S_N}{T_N}$ as a polynomial
	So $\frac{S_N}{T_N}$ is an integer because all terms are integers	A1	Justifies expression being integer
		2	
7(iv)	$\frac{S_N}{N^3T_N}=\frac{4(3N+4)(3N^2+16N+9)}{N^3}$	M1	Divides expression in **(iii)** by N^3 and takes limit
	$\to 4(3)(3)=36$	A1	
		2	

Q7.

4(i)	$\frac{1}{(3r+1)(3r-2)}=\frac{1}{3}\left(\frac{1}{3r-2}-\frac{1}{3r+1}\right)$	M1 A1	Finds partial fractions.
	$\sum_{r=1}^{N}\frac{1}{(3r+1)(3r-2)}=\frac{1}{3}\left(\frac{1}{1}-\frac{1}{4}+\frac{1}{4}-\frac{1}{7}+\cdots\frac{1}{3N-2}-\frac{1}{3N+1}\right)$	M1	At least 3 term including final term.
	$=\frac{1}{3}\left(1-\frac{1}{3N+1}\right)$	A1	AG
		4	
4(ii)	$\sum_{r=N+1}^{N^2}\frac{N}{(3r+1)(3r-2)}=\sum_{r=1}^{N^2}\frac{N}{(3r+1)(3r-2)}-\sum_{r=1}^{N}\frac{N}{(3r+1)(3r-2)}$	M1	Uses $\sum_{r=N+1}^{N^2}=\sum_{r=1}^{N^2}-\sum_{r=1}^{N}$
	$=\frac{N}{3}-\frac{N}{3(3N^2+1)}-\left(\frac{N}{3}-\frac{N}{3(3N+1)}\right)$	M1	Applies **(i)**
	$=\frac{N}{3(3N+1)}-\frac{N}{3(3N^2+1)}=\frac{N^3-N^2}{(3N+1)(3N^2+1)}$	A1	Allow simplification to common denominator.
	$\to\frac{1}{9}$ as $N\to\infty$	B1	
		4	