Vectors 2 MS

Q1.

8	Finds normal to Π_1.	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ 1 & -1 & -2 \end{vmatrix} = \mathbf{i}+3\mathbf{j}-\mathbf{k}$	M1A1		
	Finds Cartesian equation.	Equation of Π_1 : $x+3y-z=12$	A1	3	
	Finds angle between normals, using scalar product.	$\cos\theta = \left\lvert\dfrac{2-3-1}{\sqrt{11}\sqrt{6}}\right\rvert$	M1		
		$= \dfrac{2}{\sqrt{66}} \Rightarrow \theta = 75.7°$ or 1.32 rad.	A1	2	
	Finds direction of line of intersection, using vector product.	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & -1 \\ 2 & -1 & 1 \end{vmatrix} = 2\mathbf{i}-3\mathbf{j}-7\mathbf{k}$	M1A1		
	Finds point common to both planes.	Point on both planes is e.g. (6,2,0)	M1A1		
	States vector equation.	$\mathbf{r} = 6\mathbf{i}+2\mathbf{j}+t(2\mathbf{i}-3\mathbf{j}-7\mathbf{k})$ (OE)	A1	5	**[10]**

Q2.

8	Finds normal	$\overrightarrow{AB} = \mathbf{i}+2\mathbf{j}+2\mathbf{k}$, $\overrightarrow{AC} = -2\mathbf{i}+\mathbf{j}-2\mathbf{k}$			
	and cartesian equation	$\overrightarrow{AB} \times \overrightarrow{AC} = -6\mathbf{i}-2\mathbf{j}+5\mathbf{k}$	M1A1		
		$6x+2y-5z = \text{constant} = 24+10-30 = 4$	M1A1	**(4)**	
	Finds where *OD* meets plane	Equation of *OD*: $\mathbf{r} = t(6\mathbf{i}+3\mathbf{j}+6\mathbf{k})$	B1		
		$\Rightarrow 36t+6t-30t = 12t = 4 \Rightarrow t = \dfrac{1}{3}$	M1A1		
		E is the point (2,1,2).	A1	**(4)**	
	Obtains angles between planes	Using $(6\mathbf{i}+2\mathbf{j}-5\mathbf{k}).2\mathbf{i}+\mathbf{j}+2\mathbf{k}$			
		$\Rightarrow 12+2-10 = \sqrt{36+4+25}\sqrt{4+1+4}\sin\theta$	M1A1		
		$\Rightarrow \theta = 9.5°$ (0.166 rad).	A1	**(3)**	**[11]**

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Q3.

11	$\overrightarrow{AB}=\begin{pmatrix}6\\4\\-8\end{pmatrix}$ $\overrightarrow{CD}=\begin{pmatrix}5\\-2\\-4\end{pmatrix}$	B1
	Common perpendicular is $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\3 & 2 & -4\\5 & -2 & -4\end{vmatrix}=\begin{pmatrix}-16\\-8\\-16\end{pmatrix}\sim\begin{pmatrix}2\\1\\2\end{pmatrix}$	M1A1
	$\overrightarrow{AD}=\begin{pmatrix}1\\-4\\9\end{pmatrix}\Rightarrow$ shortest distance $=\left\lvert\dfrac{\begin{pmatrix}1\\-4\\9\end{pmatrix}\cdot\begin{pmatrix}2\\1\\2\end{pmatrix}}{\sqrt{4+1+4}}\right\rvert=\dfrac{16}{3}$ or 5.33	M1A1 [5]
	Normal to Π_1 is $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & -4 & 9\\3 & 2 & -4\end{vmatrix}=\begin{pmatrix}-2\\31\\14\end{pmatrix}$	M1A1
	Normal to Π_2 is $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & -4 & 9\\5 & -2 & -4\end{vmatrix}=\begin{pmatrix}34\\49\\18\end{pmatrix}$	A1
	$\cos\theta=\dfrac{-2\times34+31\times49+14\times18}{\sqrt{2^2+31^2+14^2}\sqrt{34^2+49^2+18^2}}$	M1A1√
	$\Rightarrow\theta=36.7°$ (CAO)	A1 [6]

Q4.

	OR	
(i)	$\mathbf{r}=4\mathbf{i}-2\mathbf{j}+2\mathbf{k}+\lambda(\mathbf{i}+7\mathbf{j}+\mathbf{k})+\mu(3\mathbf{i}+\mathbf{j}-\mathbf{k})\Rightarrow A$ is in Π_1.	B1 [1]
(ii)	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & 7 & 1\\3 & 1 & -1\end{vmatrix}=\begin{pmatrix}-8\\4\\-20\end{pmatrix}\sim\begin{pmatrix}2\\-1\\5\end{pmatrix}$	M1A1 [2]
(iii)	L is (12, –6, 6) $2x-y+5z=24+6+30=60$ $\mathbf{n}=t(2\mathbf{i}-\mathbf{j}+5\mathbf{k})\Rightarrow 4t+t+25t=60\Rightarrow t=2$ $\mathbf{n}=4\mathbf{i}-2\mathbf{j}+10\mathbf{k}$	B1 B1 M1A1√ A1 [5]
(iv)	$\mathbf{m}=4\mathbf{i}-2\mathbf{j}+2\mathbf{k}+\frac{3}{4}(8\mathbf{i}-4\mathbf{j}+4\mathbf{k})=10\mathbf{i}-5\mathbf{j}+5\mathbf{k}$ (AG)	B1
	M is (10, –5, 5) $\Rightarrow\overrightarrow{NM}=6\mathbf{i}-3\mathbf{j}-5\mathbf{k}$ $(6\mathbf{i}-3\mathbf{j}-5\mathbf{k})\times(2\mathbf{i}-\mathbf{j}+5\mathbf{k})=-\mathbf{i}+2\mathbf{j}$ Perpendicular distance is $\dfrac{\lvert 20(-\mathbf{i}+2\mathbf{j})\rvert}{\sqrt{30}}=\dfrac{20}{\sqrt{6}}=8.16$ (Mark various alternative methods in a similar manner.)	B1√ M1A1 M1A1 [6]

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Q5.

10 (i)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & -3 \\ -2 & 1 & 3 \end{vmatrix} = \begin{pmatrix} -3 \\ 3 \\ -3 \end{pmatrix} \sim \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	M1A1
	$\overrightarrow{BA} = 6\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$	B1
	Shortest distance $\left\| \frac{\begin{pmatrix} 6 \\ 4 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}}{\sqrt{1^2+1^2+1^2}} \right\| = \frac{4}{\sqrt{3}}$ $(= 2.31)$	M1A1 (5)
	Alternative	
	$\begin{pmatrix} -6-2\lambda-\mu \\ -4+\lambda+2\mu \\ 6+3\lambda+3\mu \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} = 0 \Rightarrow -13\lambda - 14\mu = 16$	(M1)
	$\begin{pmatrix} -6-2\lambda-\mu \\ -4+\lambda+2\mu \\ 6+3\lambda+3\mu \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} = 0 \Rightarrow 14\lambda + 113\mu = -26$	(A1)
	$\Rightarrow \lambda = -\frac{52}{9},\ \mu = \frac{38}{9}$	(M1A1)
	Shortest distance $= \frac{4}{\sqrt{3}}$. $(= 2.31)$	(A1)
(ii)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ -2 & 1 & 3 \end{vmatrix} = \begin{pmatrix} -4 \\ -5 \\ -1 \end{pmatrix} \sim \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix}$	M1A1
	Cartesian equation of Π: $4x + 5y + z = -12 - 5 + 2 = -15$	M1A1 (4)
(iii)	Distance of A from Π: $\left\| \frac{\begin{pmatrix} 6 \\ 4 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix}}{\sqrt{4^2+5^2+1^2}} \right\|$	M1A1
	$= \frac{38}{\sqrt{42}}$ $(= 5.86)$	A1 (3) **[12]**

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Q6.

8	$\begin{pmatrix} 3+2\lambda \\ \lambda \\ 2-2\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 3$	M1
	$3+2\lambda+2\lambda+2-2\lambda=3 \Rightarrow \lambda=-1$	A1
	P is $(1, -1, 4)$ (Accept position vector.)	A1 (3)
	$\mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$	B1
	$\mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 1 \\ 2 & 1 & -2 \end{vmatrix} = \begin{pmatrix} -5 \\ 4 \\ -3 \end{pmatrix}$	M1A1 (3)
	Direction of PQ is $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 4 & -3 \\ 1 & 2 & 1 \end{vmatrix} = \begin{pmatrix} 10 \\ 2 \\ -14 \end{pmatrix} \sim \begin{pmatrix} 5 \\ 1 \\ -7 \end{pmatrix}$ (OE)	M1A1
	Equation of PQ is $\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 1 \\ -7 \end{pmatrix}$	dM1A1 (4)
	[N.B. For methods not using **w** at most three B1 marks.]	**Total 10**

Vectors 2 MS

Q7.

8	$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 3 & -8\\ -3 & 4 & -5\end{vmatrix}=\begin{pmatrix}17\\34\\17\end{pmatrix}\sim\begin{pmatrix}1\\2\\1\end{pmatrix}$	**M1 A1**
	So $x+2y+z=\text{const}\Rightarrow \text{const}=2-2+3=3\,(\text{using a point})\Rightarrow$ $x+2y+z=3$	**M1** **A1** [4]
	$\sqrt{9+1+4}\sqrt{1+4+1}\,\cos\theta=\begin{pmatrix}1\\2\\1\end{pmatrix}.\begin{pmatrix}3\\-1\\2\end{pmatrix}\Rightarrow\cos\theta=\frac{3}{\sqrt{14}\sqrt{6}}=\frac{3}{\sqrt{84}}$	**M1 M1**
	$\Rightarrow\theta=70.9^\circ$ or 1.24 radians	**A1** [3]
	Direction of line of intersection is $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 2 & 1\\ 3 & -1 & 2\end{vmatrix}=\begin{pmatrix}5\\1\\-7\end{pmatrix}$	**M1A1**
	Finds point common to both planes is $(-1,0,4)$ or $\left(\frac{13}{7},\frac{4}{7},0\right)$ or $(0,\frac{1}{5},\frac{13}{5})$	**M1**
	Equation of line of intersection is eg $\mathbf{r}=\begin{pmatrix}-1\\0\\4\end{pmatrix}+t\begin{pmatrix}5\\1\\-7\end{pmatrix}$.	**A1**√ [4]

Vectors 2 MS

Q8.

11 (o) (i)	Direction perpendicular to AB and CD: $\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 1 \\ 4 & 3-\lambda & 1 \end{vmatrix} = \begin{pmatrix} \lambda-2 \\ 4 \\ -4 \end{pmatrix}$	M1A1
	$\overrightarrow{DB} = \begin{pmatrix} 5 \\ -4 \\ -6 \end{pmatrix}$. Hence $\left\lvert \dfrac{\begin{pmatrix} 5 \\ -4 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} \lambda-2 \\ 4 \\ -4 \end{pmatrix}}{\sqrt{(\lambda-2)^2+16+16}} \right\rvert = 3$ or equivalent	M1A1
(ii)	$\Rightarrow (5\lambda-2)^2 = 9(\lambda^2-4\lambda+36)$ $\Rightarrow \ldots \Rightarrow \lambda^2+\lambda-20=0$ **(AG)**	M1M1 A1 [7]
	$(\lambda+5)(\lambda-4)=0 \Rightarrow \lambda=-5,4$	B1
	$\lambda=4 \Rightarrow \mathbf{a} = \begin{pmatrix} 2 \\ 4 \\ -3 \end{pmatrix} \Rightarrow$ Normal to $ABD = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 1 \\ -1 & 3 & 7 \end{vmatrix} = \begin{pmatrix} -10 \\ -29 \\ 11 \end{pmatrix}$	M1A1
	$\lambda=-5 \Rightarrow \mathbf{a} = \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} \Rightarrow$ Normal to $ABD = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 8 & 1 \\ -1 & 12 & 7 \end{vmatrix} = \begin{pmatrix} 44 \\ -29 \\ 56 \end{pmatrix}$	A1
	$\cos\theta = \dfrac{-440+841+616}{\sqrt{10^2+29^2+11^2}\sqrt{44^2+29^2+56^2}} = \dfrac{1017}{\sqrt{1062}\sqrt{5913}}$	M1A1
	$\Rightarrow \theta = 66.1°$	A1 [7]